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Dimensional traps in evasion models and their effects on industrial structure ☆

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ABSTRACT

Size-dependent firms' monitoring by the state leads to sub-optimal results in terms of combating evasion and efficient allocation of investment by firms. While Coppier, Michetti and Scaccia (2022) describe how such a policy can be a source of dimensional trap in a single-firm evasion model, the present paper moves to a heterogeneous context, considering industrial structures composed of many firms with different sizes. We then propose a discrete-time nondeterministic dynamic model to describe the consequences of size-dependent policies and potential dimensional traps on industrial structure and its evolution over time. We show that an unwise choice of policy parameters may determine a long-run equilibrium industrial structure characterized by a small number of large firms and a plethora of small firms, with the latter being marked by inefficient resource allocation and noncompliant behavior with regard to tax regulations. These results are robust to different choices of the initial industrial structure suggesting alternative policy indications.

1. Introduction and stylized facts

All modern economies, in both developed and developing countries, collect taxes to finance the provision of collective public goods and services such as transportation, defense, health care, and education systems. However, nearly all countries have a high level of tax evasion, i.e., noncompliance with tax laws. Prevalence of evasion refers to the phenomenon whereby individuals or entities engage in practices aimed at avoiding, minimizing, or evading their legal obligations, particularly in the areas of taxation, financial reporting, and regulatory compliance. The impact of tax evasion on the economy is profound and multifaceted. This behavior undermines the effective functioning of economies, tax systems, and regulatory frameworks. A direct consequence of tax evasion is a loss of tax revenue to the government. When taxpayers evade taxes, the government is deprived of capital that could fund public services, infrastructure development, education, health care, and other essential programs. This reduction in revenue can hinder the government's ability to meet the needs of citizens and lead to budget deficits. In addition, widespread tax evasion can erode citizens' trust in government institutions. Finally, tax evasion can distort economic decision-making. Indeed, when a firm evades, this can lead to the nonoptimal allocation of resources, reducing economic growth.[1]

While precisely quantifying the extent of tax evasion is not easy, there is common agreement that tax evasion is a quantitatively relevant phenomenon even in developed countries. In Europe, for example, tax evasion is estimated to account for around 20% of GDP, equivalent to a potential loss of around 1 trillion euros each year (Buehn and Schneider, 2012). Murphy (2019) estimates that the tax gap for 2015, i.e., the tax loss, in the European Union was about 825 billion euros. Similarly, the VAT tax gap, the only tax gap for which there are comparative estimates for all EU countries, was estimated to be around 137.5 billion euros in 2017 (Poniatowski et al., 2022).

Starting from the pioneering work of Allingham and Sandmo (1972), the "classic" tools to combat tax evasion identified by the literature are the enforcement and sanctioning of tax evaders.[2] To

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[1] For an analysis of the effects of evasion on output and economic growth, see, e.g., Coppier and Michetti (2006) and Cerqueti and Coppier (2011).

[2] In addition to the classic tools to combat evasion, recent literature, both empirical and theoretical, finds that the decision to evade is shaped by nonpecuniary factors, i.e. moral and social considerations (see, for example, Bethencourt and Kunze, 2020; Luttmer and Singhal, 2014). Therefore, a nonpecuniary cost, formalized as a social norm towards tax compliance (tax morale) can also represent a tool to combat evasion.

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help countries devise effective and efficient strategies to combat evasion, some major international organizations (e.g., the International Monetary Fund) provide technical assistance to member countries to improve revenue collection and enforcement.[3] One proposed strategy is to put different procedures in place for different segments of taxpayers, with particular attention often paid to large taxpayers. The expected advantages of establishing Large Taxpaying Units (LTUs) are mainly identified as the greater ability to obtain tax revenue and, more generally, the strengthening of tax system efficiency. To segment taxpayers, the tax authority generally sets arbitrary thresholds. In this case, taxpayers often face similar tax structures but are likely to be subjected to different audit procedures related to the arbitrary threshold that was set: the tax authority puts different audit probabilities in place for firms of different sizes.

Despite the wide diffusion of these size-dependent tax enforcement strategies, micro- and macro-level evidence on the effects of LTUs on firms' behavior is very sparse. Some authors (see, for example Almunia and Lopez-Rodriguez, 2018; Bachas et al., 2019; Boonzaaier et al., 2019; Gourio and Roys, 2014; Guner et al., 2008; López and Torres, 2020; Onji, 2009; Ramaswamy, 2021) warn about the risk that firms may intentionally opt to remain small to avoid stricter regulations. Consequently, size-dependent enforcement could inadvertently encourage underreporting of revenue or even hinder growth. In particular, such regulations risk causing a misallocation of resources: the most efficient firms might lack an incentive to grow, resulting in a significant decline in average firm size and a decrease in innovation. In line with these concerns, Coppier et al. (2023) consider a one-firm framework and use a nondeterministic dynamic model to derive the conditions under which a firm may find it convenient not to invest and remain small to avoid a higher level of enforcement (dimensional trap). The present work aims to extend the analysis to a multiple-firm framework and to identify the consequences of size-dependent regulations on the behavior of firms (their tax compliance, and their decision to invest and grow) and on the resulting industrial structure. We thus relax the single-firm assumption of Coppier et al. (2023) and consider heterogeneous firms in terms of size (approximated by levels of capital). Endogenizing audit probability by firm size and assuming two different levels of control for small and large firms, we develop a dynamic model to evaluate the effects of this size-dependent regulation. Simulating the temporal evolution of this dynamic model, we analyze and evaluate the different firms' behaviors and their effect on the spread of tax evasion, investment growth, and the consequent industrial structure. In practice, depending on the probability of an audit and the fine for detected evasion, each firm must decide whether to evade taxes and, if so, whether to limit its investments in order to remain small and face less monitoring. Firm decisions about compliance and capital accumulation determine the overall level of evasion in the long run, as well as the growth and structure of the industrial system. The analysis is carried out for different possible starting scenarios of the industrial structure to evaluate whether size-dependent fiscal policies may produce different outcomes depending on the initial conditions. Various policy parameter values are also considered, and their effects on compliance levels and capital accumulation dynamics are evaluated to provide useful regulation guidelines.

The framework of this paper is motivated and supported by some stylized facts, at both the micro and the macro level, concerning firm size, audit probability, tax evasion, industrial structure, and the relationships among those factors. At the micro level, the data show that smaller firms are less likely to be subject to a tax audit than larger companies. In this regard, it is interesting to consider, as documented in Coppier et al. (2022, 2023), the emblematic case of Italy, where in 2018, based on the report of the CGIA-MESTRE (2019), micro and small enterprises have a 3% probability of being inspected, compared with 14% and to 32%, respectively, for medium-sized and large companies. The fact that smaller firms are less likely to be audited translates, at the macro level, into the stylized fact that there is a relationship between evasion rates and industrial structure. Considering the stylized facts above, we develop our model from crucial evidence at the micro level that, ceteris paribus, large firms are more likely to be audited by tax authorities than small firms. Starting from this micro-level assumption, the model allows us to acquire relevant information on the macro-level relationship between the diffusion of tax evasion and the industrial structure, as channeled by the size-dependent enforcement regulations. Therefore, this article extends the analysis done in Coppier et al. (2023) by considering a heterogeneous set of firms and showing how micro-level choices by firms concerning tax compliance and investment strategies (dimensional trap) could influence the spread of evasion and configuration of the industrial structure.

Our results indicate that, depending on the choice of policy parameters (namely the audit probabilities of small and large firms and the fines imposed in cases of detected evasion), size-dependent enforcement may prove ineffective in combating tax evasion and could inadvertently incentivize firms to stay small to avoid heightened scrutiny. In the long run, this could lead to the formation of an industrial structure dominated by a handful of large firms and numerous small ones marked by inefficient resource allocation and noncompliance with fiscal regulations. Our study also reveals how fines imposed on tax evaders serve as a crucial tool to prevent these distortions.

The paper is organized as follows. Section 2 recaps the single-firm model and extends it to the multiple-firm framework. Section 3 describes the main findings from the analytical results and numerical evidence. Section 4 concludes and suggests possible further development.

2. Evasion model with many firms

2.1. Single firm setting: a recap

This section summarizes the main ingredients of the model, considering just a single firm (see Coppier et al., 2023, for more details), before moving on to the multiple-firm context.

Let us consider a system composed of one representative firm in a discrete-time setup, i.e. $t \in \mathbb{N}$. To describe capital-per-capita and monitoring level evolution, we consider the following assumptions:

- When producing, the firm can face a bad or good state of nature, i.e. $SN_t = \{0, 1\}$ where SN_t is a Bernoulli variable taking a value of 0 or 1, respectively, for the bad or good state, with success probability given by $\text{Prob}(SN_t = 1) = \theta \in (0, 1)$.
- Given the capital-per-capita level $k_t \geq 0$, let $f_h(k_t)$ be the production function in case of a good state of nature and $f_l(k_t)$ be the production function in case of a bad state of nature, where $f_h(k_t) > f_l(k_t) > 0$, $\forall k_t > 0$, and let $m(k_t)$ be the fine function to be paid if firm tax evasion is discovered. Taxation over firm profit is assumed to be based on an exogenous tax rate $\tau \in (0, 1)$.[4]
- The expectation of the firm about the monitoring level, $q_t \in [0, 1]$, put in place by the state, is given by $E(q_t) \in [0, 1]$; in the single-firm setup, complete information is assumed, i.e. the firm knows the monitoring level function used by the state, hence $E(q_t) = q_t$.
- The firm may find it convenient to lie about its state of nature and evade; variable $e_t = \{0, 1\}$ assumes a value of 1 if the firm evades taxes at time t and 0 otherwise.

[3] For a dynamic general equilibrium analysis that looks at the optimal mix of tax rate and control parameters for governments to raise the necessary revenue to implement optimal public spending policies, see, for example, Dzhumashev et al. (2023).

[4] Our model uses a tax proportional to profits. For a quantitative analysis comparing the effects of a progressive tax with a proportional one, see for example Fernández-Bastidas (2023).

- A firm declaring a bad state of nature can be monitored by the state. Thus, $c_t = \{0, 1\}$ denotes a Bernoulli variable with a value of 1 if the firm is monitored and 0 otherwise, with $\text{Prob}(c_t = 1) = q_t$.
- Depending on SN_t, e_t, c_t, profits π_t are realized. Let the depreciation rate of capital be given by $\delta \in [0, 1]$; the firm then decides the fraction $\mu_t \in [0, 1]$ of its realized profit to invest in production, with k_{t+1} determined by $k_{t+1} = (1 - \delta)k_t + \mu_t \pi_t$.

We recall the following result proved in Coppier et al. (2023).

Proposition 2.1. *Assume $SN_t = 1$, and let*

$$q_t^\star = 1 - \frac{m(k_t)}{\tau(f_h(k_t) - f_l(k_t)) + m(k_t)} = \frac{\tau(f_h(k_t) - f_l(k_t))}{m(k_t) + \tau(f_h(k_t) - f_l(k_t))}. \tag{1}$$

Then

- *if $E(q_t) \geq q_t^\star$, it is not profitable for the firm to evade,*
- *if $E(q_t) < q_t^\star$, it is profitable for the firm to evade.*

Regarding production function and fine function, from Coppier et al. (2023), we consider the following assumption.

Remark 2.2. *Production function and fine level are defined as follows:*

- *The production function is of the Cobb–Douglas type, given by $f_h(k_t) = A_h k_t^\alpha$ and $f_l(k_t) = A_l k_t^\alpha$, with $A_h > A_l > 0$ and $\alpha \in (0, 1)$.*
- *The fine is assumed to be proportional to the profit realized in case of undetected evasion and given by $m(k_t) = m_0(f_h(k_t) - \tau f_l(k_t))$, where the positive constant m_0 is the strength of the fine. We assume that the fine to be paid cannot exceed the total amount of realized profit so that profit cannot become negative. Hence, $(1-\tau)A_h k_t^\alpha - m_0(A_h k_t^\alpha - \tau A_l k_t^\alpha) \geq 0$, so the following relation holds:*

$$0 < m_0 \leq \frac{(1 - \tau)A_h}{A_h - \tau A_l} = m_0^M. \tag{2}$$

As a consequence the following Proposition trivially holds.

Proposition 2.3. *Let $f_h(k_t) = A_h k_t^\alpha$, $f_l(k_t) = A_l k_t^\alpha$ and $m(k_t) = m_0(A_h k_t^\alpha - \tau A_l k_t^\alpha)$. The threshold level is then*

$$q_t^\star = q^\star = \frac{\tau(A_h - A_l)}{m_0(A_h - \tau A_l) + \tau(A_h - A_l)}, \tag{3}$$

which is constant for all $k_t \geq 0$.

As discussed in the introduction, to consider that small firms are subject to a lower monitoring level than large firms, the function q_t follows the rule:

$$q_t = \Phi(k_t) := \begin{cases} q_l \text{ if } k_t \in [0, k^\star] \\ \\ q_h \text{ if } k_t > k^\star \end{cases} \tag{4}$$

with $k^\star > 0$ and $0 \leq q_l \leq q_h \leq 1$. Such an assumption follows Almunia and Lopez-Rodriguez (2018) and considers that the probability of audit jumps up discretely at a given level of declared profits, which we approximate by capital level. Here $k^\star$ represents the threshold separating the high monitoring regime from the low monitoring one and it is assumed to be constant.

2.2. Moving to the multiple-firm setting

As modeled in Coppier et al. (2023), and summarized in the previous section, the single-firm perspective assumes that the monitoring level attached by the state to the firm only depends on the capital-per-capita level of that firm regardless of the dimensions of other firms. The main novelty of this work is to consider the whole industrial structure, made up of several firms with different dimensions. As a consequence, we revise the mechanism the state uses to fix the monitoring level, $k^\star$, for a multi-firm framework, a crucial step for extending the analysis of Coppier et al. (2023).

To account for the role of firms' dimensions, we allow firms to have different capital-per-capita levels, so the jth firm has a $k_{j,t} \geq 0$ capital-per-capita level at time t for $j = 1, \ldots, N$, with N being the number of firms in the market.[5] We then assume that large firms are more likely than small firms to be monitored. Given that the dimension of each firm changes over time, the dimensional threshold $k^\star$ must be specified by accounting for evolution of the industrial structure. Let $\gamma \in [0, 1]$ be the monitoring effort the state put in fighting evasion (related, for instance, to the amount of resources devoted to the scope). In the present formulation, γ is supposed to be exogenous and constant for all t, and represents the proportion of firms the state wants to monitor with higher probability. The new assumption introduced here considers that the state fixes the dimensional threshold $k_t^\star$, now dependent on time, in such a way that, after sorting the firms in ascending order according to their capital-per-capita level $k_{j,t}$, γN firms have capital-per-capita no larger than $k_t^\star$. Hence, the threshold value changes over time with the firms' dimensions, thus assuring that the γN largest firms in the market face the highest probability of being monitored. According to such arguments Eq. (4) is revised and the following assumption is introduced.

Remark 2.4. *Let $k_t^\star$ be such that γN firms have $k_{j,t} > k_t^\star$. The monitoring level, $q_{j,t}$, faced by firm j, is then given by:*

$$q_{j,t} = \Phi(k_{j,t}) := \begin{cases} q_l \text{ if } k_{j,t} \in [0, k_t^\star] \\ \\ q_h \text{ if } k_{j,t} > k_t^\star \end{cases}. \tag{5}$$

Once each firm j has concluded production at time t, decided whether to evade taxes, been monitored or not by the state, and paid any applicable fine, it realizes its profits $\pi_{j,t}$. The investment choice must then be considered, as it determines the new level of capital-per-capita available for production at time $t+1$, i.e. $k_{j,t+1}$. It should be noted that firms decide how much to invest at time t, based on the expected profit in $t + 1$. However, to calculate the expected profit in $t + 1$ they use the threshold $k_t^\star$, as they cannot predict in any way the value $k_{t+1}^\star$, which will be fixed by the state only when all $k_{j,t+1}$ are known.

Considering Eq. (5), some firms may find it convenient not to invest all their realized profits so they do not exceed the critical threshold $k_t^\star$ and therefore face a lower probability of being monitored. Hence, each firm determines $\mu_{j,t} \in [0, 1]$, representing the fraction of realized profits to invest in production, by solving a constrained maximization problem. Assuming that the capital depreciates at a constant rate $\delta \in [0, 1]$, the capital-per-capita level available for production at time $t + 1$ is

$$k_{j,t+1} = (1 - \delta)k_{j,t} + \mu_{j,t}\pi_{j,t},$$

which is bounded as follows:

$$k_{j,t+1} \in [k_{j,t+1}^m, k_{j,t+1}^M], \quad k_{j,t+1}^m = (1 - \delta)k_{j,t}, \quad k_{j,t+1}^M = (1 - \delta)k_{j,t} + \pi_{j,t}.$$

Let $\mu_{j,t}^\star \in [0, 1]$ be the solution of the constrained maximization problem, i.e., the fraction of profits the firm j invests at time t to contribute to production at time $t + 1$. If $\mu_{j,t}^\star < 1$, a situation emerges, defined as a dimensional trap, where firm j finds it convenient to not invest all

[5] Our model uses the per-worker capital level as a proxy for firm size: as the per-worker capital level increases, firm size also increases. This assumption can be justified because the per-worker capital level can be an indicator of the efficiency with which a firm uses its capital resources to generate output. Consequently, higher per-worker capital level may suggest higher labor productivity and thus better labor utilization by the firm.

its realized profits.[6] To simplify the notation, we define the following parameter combinations:

$$H = (1-\tau)A_l + \theta(A_h - A_l),$$

$$J = \theta[(1-\tau)A_h - (A_h - \tau A_l)(m_0+1)],$$

$$L = (1-\theta)(1-\tau)A_l + \theta(1-\tau)A_h.$$

The following proposition establishes the condition for a dimensional trap to emerge.

Proposition 2.5. *Define*

$$g_1(k_{j,t+1}) := \begin{cases} g_{1l}(k_{j,t+1}) = k^{\alpha}_{j,t+1}(H + Jq_l) \text{ if } k_{j,t+1} \in [0, k^{\star}_t] \\ g_{1h}(k_{j,t+1}) = k^{\alpha}_{j,t+1}(H + Jq_h) \text{ if } k_{j,t+1} > k^{\star}_t \end{cases}, \quad (6)$$

$$g_2(k_{j,t+1}) := \begin{cases} g_{2l}(k_{j,t+1}) = k^{\alpha}_{j,t+1}(H + Jq_l) \text{ if } k_{j,t+1} \in [0, k^{\star}_t] \\ g_{2h}(k_{j,t+1}) = k^{\alpha}_{j,t+1}L \text{ if } k_{j,t+1} > k^{\star}_t \end{cases}, \quad (7)$$

and

$$g_3(k_{j,t+1}) = (1-\theta)(1-\tau)f_l(k_{j,t+1}) + \theta(1-\tau)f_h(k_{j,t+1}) = k^{\alpha}_{j,t+1}L. \quad (8)$$

1. *Let $q^\star \in (q_h, 1]$. If $k^m_{j,t+1} \le k^\star_t < k^M_{j,t+1}$ and $g_{1l}(k^\star_t) > g_{1h}(k^M_{j,t+1})$ then $k_{j,t+1} = k^\star_t$ and $\mu^\star_{j,t} < 1$. Else, $k_{j,t+1} = k^M_{j,t+1}$ and $\mu^\star_{j,t} = 1$.*
2. *Let $q^\star \in (q_l, q_h]$. If $k^m_{j,t+1} \le k^\star_t < k^M_{j,t+1}$ and $g_{2l}(k^\star_t) > g_{2h}(k^M_{j,t+1})$ then $k_{j,t+1} = k^\star_t$ and $\mu^\star_{j,t} < 1$. Else, $k_{j,t+1} = k^M_{j,t+1}$ and $\mu^\star_{j,t} = 1$.*
3. *Let $q^\star \in [0, q_l]$. Then $k_{j,t+1} = k^M_{j,t+1}$ and $\mu^\star_{j,t} = 1$.*

Proof. Let $q^\star$ as defined in (3) and $q_{j,t+1} = \Phi(k_{j,t+1})$ as specified in (5). We distinguish between the following cases.

1. Assume $q^\star \in (q_h, 1]$; then $E(q_{j,t+1}) < q^\star$, and evasion is expected to be profitable for all $k_{j,t+1}$. The expected profit under evasion is given by

$$\begin{aligned} E(\pi_{j,t+1}) &= (1-\theta)(1-\tau)f_l(k_{j,t+1}) + \theta\{\Phi(k_{j,t+1})[(1-\tau)f_h(k_{j,t+1}) \\ &\quad - m(k_{j,t+1})] + (1-\Phi(k_{j,t+1}))(f_h(k_{j,t+1}) - \tau f_l(k_{j,t+1}))\} \\ &= k^{\alpha}_{j,t+1}(H + J\Phi(k_{j,t+1})) = g_1(k_{j,t+1}). \end{aligned}$$

   Consider $q_h \in [0,1]$, $h = \{l, q\}$. Since condition (2) holds,

$$\begin{aligned} H + Jq_h &= (1-\tau)A_l + \theta(A_h - A_l) + \theta q_h[(1-\tau)A_h \\ &\quad - (A_h - \tau A_l)(m_0+1)] \\ &> (1-\tau)A_l + \theta(A_h - A_l) + \theta q_h\left[(1-\tau)A_h - (1-\tau)A_h \right.\\ &\quad \left. -(A_h - \tau A_l)\right] \\ &= (1-\tau)A_l + \theta(A_h - A_l) - \theta q_h(A_h - \tau A_l) \\ &> (1-\tau)A_l + \theta(A_h - A_l) - \theta(A_h - \tau A_l) \\ &= A_l(1-\tau)(1-\theta) > 0 \end{aligned}$$

   Hence both g_{1l} and g_{1h} are strictly increasing functions. The following cases may occur: (1.1) if $k^M_{j,t+1} \le k^\star_t$, $\max\{E(\pi_{j,t+1})\} = g_{1l}(k^M_{j,t+1})$, and $\mu^\star_{j,t} = 1$ — that is, all profits are invested; (1.2) if $k^m_{j,t+1} \le k^\star_t < k^M_{j,t+1}$, being that $g_{1l}(k^\star_t) > \lim_{k_{j,t+1}\to k^{\star +}_t} g_{1h}(k_{j,t+1})$, the constrained expected payoff maximization implies $\max\{E(\pi_{j,t+1})\} = \max\{g_{1l}(k^\star_t), g_{1h}(k^M_{j,t+1})\}$, so the dimensional trap occurs whenever $g_{1l}(k^\star_t) > g_{1h}(k^M_{j,t+1})$; (1.3) if $k^\star_t < k^m_{j,t+1}$, $\max\{E(\pi_{j,t+1})\} = g_{1h}(k^M_{j,t+1})$, and $\mu^\star_{j,t} = 1$ — that is, all profits are invested.
2. Assume $q^\star \in (q_l, q_h]$. The expected payoff can be written as $E(\pi_{j,t+1}) = g_2(k_{j,t+1})$. The following cases may occur: (2.1) if $k^M_{j,t+1} \le k^\star_t$, according to Proposition 2.1, evasion is ex-ante convenient for all $k_{j,t+1}$ and the maximum expected payoff corresponds to $\mu^\star_{j,t} = 1$; (2.2) if $k^m_{j,t+1} \le k^\star_t < k^M_{j,t+1}$, both g_{2l} and g_{2h} are strictly increasing functions in their domain, so the constrained maximization problem solution is the one associated with the higher value between $g_{2l}(k^\star_t)$ and $g_{2h}(k^M_{j,t+1})$; (2.3) if $k^\star_t < k^m_{j,t+1}$ and the maximum expected payoff corresponds to $\mu^\star_{j,t} = 1$.
3. Assume $q^\star \in [0, q_l]$; then $E(q_{j,t+1}) \ge q^\star$, and according to Proposition 2.1, evasion is not convenient for any $k_{j,t+1}$. The expected profit without evasion is given by $E(\pi_{j,t+1}) = g_3(k_{j,t+1})$, which is continuous, strictly increasing, and independent of the monitoring level. As a consequence, $\max\{E(\pi_{j,t+1})\} = g_3(k^M_{j,t+1})$ and $\mu^\star_{j,t} = 1$.

   □

According to Proposition 2.5, a dimensional trap may arise so long as $q^\star > q_l$ if $k^\star_t \in [k^m_{j,t+1}, k^M_{j,t+1})$. In fact, in the case where $q^\star > q_h$, all firms find it convenient to evade. Some firms face a high probability of being caught, whereas others face a low probability. Those for which $k^\star_t \in [k^m_{j,t+1}, k^M_{j,t+1})$ can decide not to invest all their profits in order to remain below the dimensional threshold and not face a higher probability of audit. In this case, the dimensional trap occurs, as these firms decide to remain small. When $q_l < q^\star \le q_h$, only small firms find convenience in evading taxes, and again, those for which $k^\star_t \in [k^m_{j,t+1}, k^M_{j,t+1})$ might find it worthwhile to continue evading and remain small to incur a lower auditing probability. Once the investment decision has been taken and the capital-per-capita updated, the story repeats, as summarized in Fig. 1.

3. Empirical study description and results

3.1. Empirical framework

Similarly to Coppier et al. (2022), to describe the evolution of the industrial structure, the evasion index, and the spread of the dimensional trap, we consider four initial market structures:

- Scenario 1: all firms have different sizes and are uniformly distributed over the size interval $[1/c, N/c]$. That is, $k_{j,0} = j/c$, for $j = 1, \dots, N$, with c being a small positive constant. We set $c = 20$ in the present study.
- Scenario 2: all firms are approximately homogeneous in size. That is, $k_{j,0} = \bar{k} + \epsilon_j$, for $j = 1, \dots, N$, with ϵ_j being a random perturbation drawn from a standard normal distribution. Without loss of generality, we assume $\bar{k} = 10$.
- Scenario 3: the market is characterized by a small number of similarly little firms and a large number of similarly big firms, which represent almost the whole market. In particular, we assume that 1% of firms have a low capital level equal to $k_{j,0} = 1 + 0.2\epsilon_j$, for $j = 1, \dots, \lfloor 0.01N \rfloor$, and that 99% of firms have a high capital level equal to $k_{j,0} = 10 + 2\epsilon_j$, for $j = \lfloor 0.01N \rfloor + 1, \dots, N$, with the symbol $\lfloor \cdot \rfloor$ indicating the floor of a real number.
- Scenario 4: the market is characterized by a large number of similarly small firms and a small number of similarly big firms. In particular, we assume that 99% of small firms have a low capital level equal to $k_{j,0} = 1 + 0.2\epsilon_j$, for $j = 1, \dots, \lfloor 0.99N \rfloor$, and that 1% of firms have a high capital level equal to $k_{j,0} = 10 + 2\epsilon_j$, for $j = \lfloor 0.99N \rfloor + 1, \dots, N$.

[6] In our model, because we consider the capital-per-capita level a proxy for firm size, we talk about not only the "dimensional trap" but also the "productivity trap". In fact, smaller firms, i.e., with a lower capital-per-capita level, may prefer to remain small and not invest all the profits realized. In this case, such firms will not experience technological improvements and will continue to produce with low productivity levels.

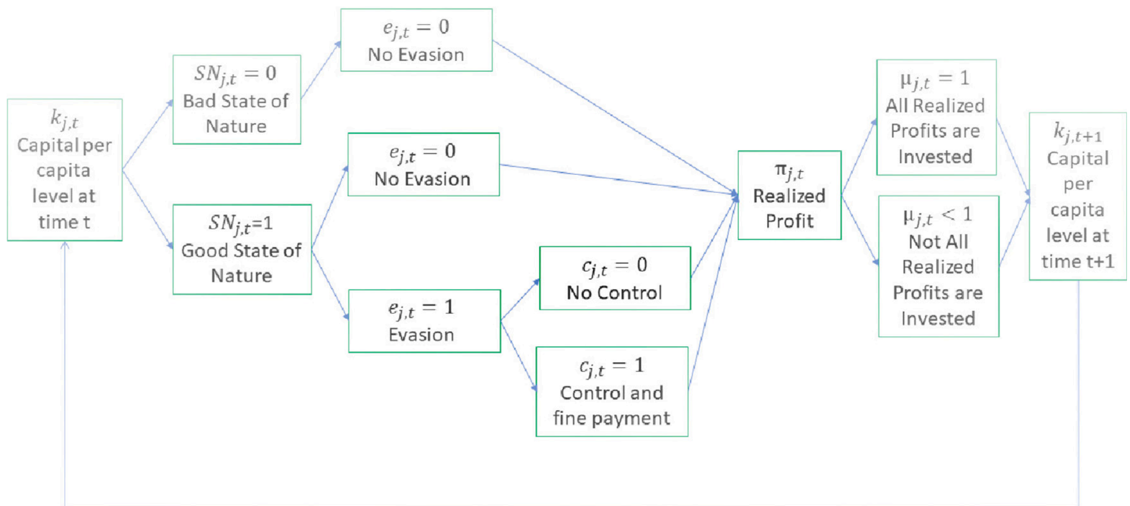

Fig. 1. Scheme of the model.

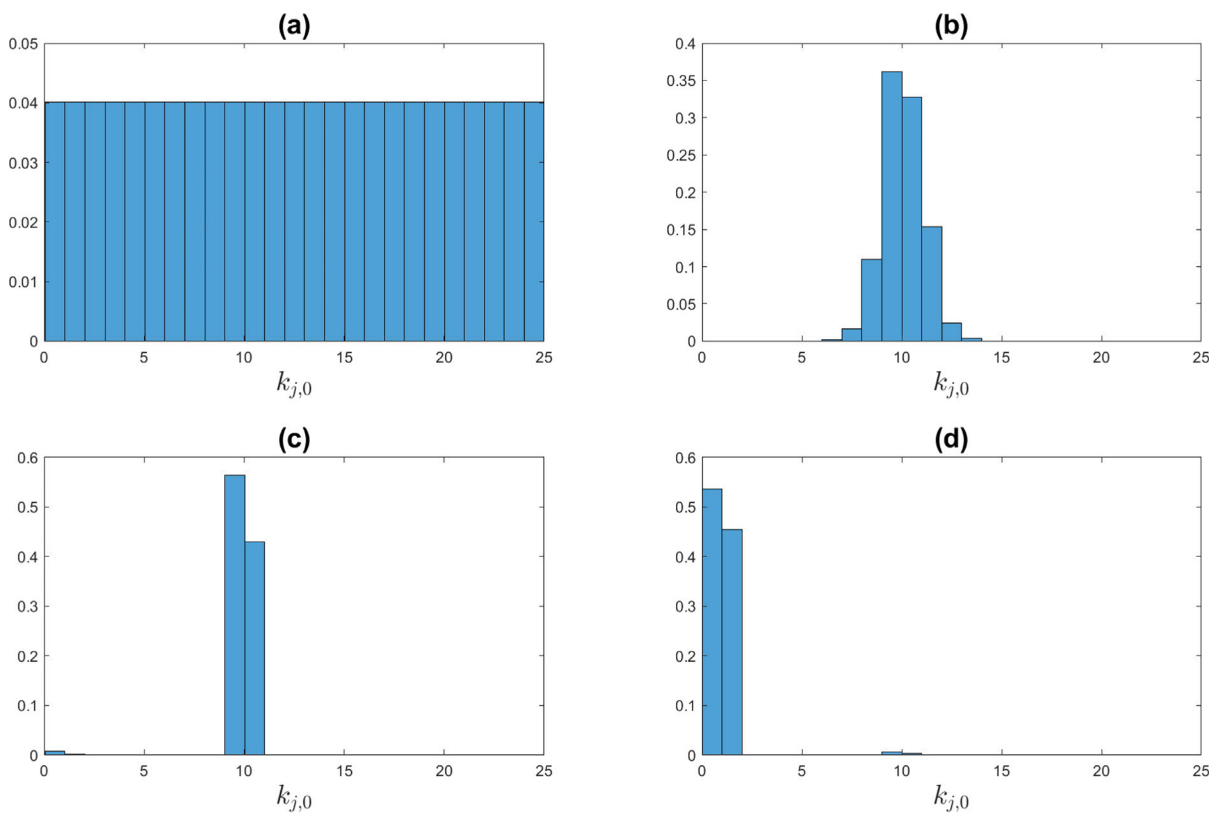

Fig. 2. Distribution of firms with respect to their starting capital-per-capita level. Four initial Scenarios are considered: (1) different and uniformly spread capital levels for all firms (2) approximately same capital level for all firms; (3) few little firms; (4) many little firms.

The four different scenarios are presented in Fig. 2 ($N = 500$ is considered). Their choice allows us to consider very different industrial structures in terms of firm size and market concentration to evaluate the effect of size-dependent tax enforcement policies under markedly dissimilar starting conditions. In fact, we consider the extreme case of an oligopoly structure,[7] characterized by high market share concentration, with a small number of firms of large size (Scenario 4), the

[7] The most extreme case of monopolistic industrial structure is provided in Coppier et al. (2022).

opposite case of perfect competition, in which all firms are very similar in terms of size and there is no concentration of market shares (Scenario 2), as well as two different intermediate cases.

The algorithm used to simulate the temporal evolution of the industrial structure closely follows the scheme described in Fig. 1. After initializing $k_{j,0}$, for $j = 1, \ldots, N$, according to one of the scenarios proposed above, the dimensional threshold at time 1 is computed as $k_0^\star = k_{[l],0}$ where $k_{[l],0}$ is the capital-per-capita level of the $l = \lfloor (1-\gamma)N \rfloor$th largest firm, the square brackets indicating that the firms have been sorted in increasing order. The following steps are then repeated for $t = 0, \ldots, T$:

1. Randomly draw the state of nature $SN_{j,t}$, $\forall j = 1, \ldots, N$, from a Bernoulli distribution with parameter θ.
2. If $SN_{j,t} = 1$ and $q_{j,t} < q^\star$, set $e_{j,t} = 1$; otherwise, set $e_{j,t} = 0$.
3. If $e_{j,t} = 0$, set $c_{j,t} = 0$; otherwise, draw a random number $u_{j,t}$ from a uniform distribution in $[0,1]$ and set $c_{j,t} = 1$ if either of the following cases occurs:
   - $k_{j,t} \le k_t^\star$ and $u_{j,t} \le q_l$,
   - $k_{j,t} > k_t^\star$ and $u_{j,t} \le q_h$.

   Otherwise, set $c_{j,t} = 0$.
4. Compute the profit in the following way:
   - if $SN_{j,t} = 0$, $\pi_{j,t} = (1-\tau)A_l k_{j,t}^\alpha$;
   - if $SN_{j,t} = 1$ and $e_{j,t} = 0$, $\pi_{j,t} = (1-\tau)A_h k_{j,t}^\alpha$;
   - if $SN_{j,t} = 1$, $e_{j,t} = 1$ and $c_{j,t} = 0$, $\pi_{j,t} = A_h k_{j,t}^\alpha - \tau A_l k_{j,t}^\alpha$;
   - if $SN_{j,t} = 1$, $e_{j,t} = 1$ and $c_{j,t} = 1$, $\pi_{j,t} = (1-\tau)A_h k_{j,t}^\alpha - m_0(A_h k_{j,t}^\alpha - \tau A_l k_{j,t}^\alpha)$.
5. Compute $k_{j,t+1}^m = (1-\delta)k_{j,t}$ and $k_{j,t+1}^M = (1-\delta)k_{j,t} + \pi_{j,t}$.
6. Set the dimensional trap indicator $d_{j,t} = 1$ and $k_{j,t+1} = k_t^\star$ if either of the following occurs:
   - $q^\star > q_h$, $k_{j,t+1}^m \le k_t^\star < k_{j,t+1}^M$ and $g_{1l}(k_t^\star) > g_{1h}(k_{j,t+1}^M)$,
   - $q_l < q^\star \le q_h$, $k_{j,t+1}^m \le k_t^\star < k_{j,t+1}^M$ and $g_{2l}(k_t^\star) > g_{2h}(k_{j,t+1}^M)$.

   Otherwise, set $d_{j,t} = 0$ and $k_{j,t+1} = k_{j,t+1}^M$.
7. Compute $k_{t+1}^\star = k_{[l],t+1}$.

Exploiting the above algorithm, we try to detect how the industrial structure, evasion dynamic, and emergence of a dimensional trap evolve over time when tax enforcement is modeled as a size-dependent mechanism. The investigation mostly considers how results are affected by the monitoring levels q_h and q_l associated, respectively, with big or small firms, the punishment level m_0 in case of evasion, and the effort γ put in place by the state in fighting evasion.

To provide a synthetic measure of the *evasion level* characterizing the economy, let $n_{j,t} = 1$ if firm j evades taxes at time t and $n_{j,t} = 0$ otherwise, and let $s_{j,t} = 1$ if firm j faces a good state of nature at time t and $s_{j,t} = 0$ otherwise. We then take the firms evading taxes over the number of firms facing a good state of nature at any time and denote it as the fraction e_t, i.e.,

$$e_t = \frac{\sum_j n_{j,t}}{\sum_j s_{j,t}} = \frac{n_t}{s_t} \in [0,1],$$

where s_t is the number of firms facing a good state of nature, while n_t is the number of those evading taxes at time t. Then, e_t is the evasion index describing the spread of evasion among firms that incur in a good state of nature. At each time t, the evasion index can be computed, so as the process reiterates, the evolution patterns of both evasion and market structure become available and can be analyzed to search for possible links between them.

To measure the spread of the dimensional trap, and, hence, how much industrial structure evolution may be affected by size-dependent enforcement, we define the ***trap index*** as follows. Let $d_{j,t} = 1$ if firm j does not invest all profits at time t and the dimensional trap takes place and $d_{j,t} = 0$ otherwise (i.e., if $\mu_{j,t} = 1$). We then take the number of firms facing a dimensional trap over the total firms and denote it as the fraction DT_t. At each time t, the ***trap index***, can be computed as

$$DT_t = \frac{\sum_j d_{j,t}}{N}$$

so as the process reiterates, the incidence of the dimensional trap in determining industrial structure evolution can be investigated.

3.2. Empirical results

Let us consider Proposition 2.5, condition 1. In this case, $q^\star \in (q_h, 1]$, always with some j for which $k_{j,t+1}^m \le k_t^\star < k_{j,t+1}^M$. As $k_t^\star$ tends to increase with time, it can be proved that it always exists (and is attainable in time) a large enough $k_t^\star$ such that, for at least some j, $g_{1l}(k_t^\star) > g_{1h}(k_{j,t+1}^M)$, and the firm finds it profitable to stay small, so $k_{j,t+1} = k_t^\star$ and $\mu_{j,t}^\star < 1$. In practice (see Equation 8 in Coppier et al., 2023, and the text that follows it), for

$$k_t^\star > \left(\frac{\left(\frac{H+Jq_l}{H+Jq_h}\right)^{1/\alpha} - (1-\delta)}{A_h - \tau A_l} \right)^{1/(\alpha-1)}, \tag{9}$$

all firms for which $k_{j,t+1}^m \le k_t^\star < k_{j,t+1}^M$ will choose not to reinvest all their profits. This situation is illustrated, for the Scenario 1, in Fig. 3, where the choice of the policy parameters leads to $q^\star = 0.638$, so $q^\star > q_h$. According to Eq. (9), and given the policy parameters, for $k_t^\star > 32.39$ all firms for which $k_{j,t+1}^m \le k_t^\star < k_{j,t+1}^M$ are caught in the dimensional trap. In addition, all firms evade taxes, whenever they are in a favorable state of nature (as $q^\star > q_{j,t}$ for all of them, at any time t).

Let us consider Proposition 2.5, condition 2. In this case, $q^\star \in (q_l, q_h]$, always with some j for which $k_{j,t+1}^m \le k_t^\star < k_{j,t+1}^M$. As $k_t^\star$ tends to increase with time, it can be proved that it always exists (and is attainable in time) a large enough $k_t^\star$ such that, for at least some j, $g_{2l}(k_t^\star) > g_{2h}(k_{j,t+1}^M)$, and the firm finds it profitable to stay small, so $k_{j,t+1} = k_t^\star$ and $\mu_{j,t}^\star < 1$. In practice (see Equation 10 in Coppier et al., 2023, and the text that follows it), for

$$k_t^\star > \left(\frac{\left(\frac{H+Jq_l}{L}\right)^{1/\alpha} - (1-\delta)}{A_h - \tau A_l} \right)^{1/(\alpha-1)}, \tag{10}$$

all firms for which $k_{j,t+1}^m \le k_t^\star < k_{j,t+1}^M$ will choose not to reinvest all their profits. This situation is illustrated for Scenario 1, in Fig. 4, where the choice of policy parameters leads to $q^\star = 0.638$, so $q^\star \in (q_l, q_h]$. According to Eq. (10), and given the policy parameters, for $k_t^\star > 15.43$ all firms for which $k_{j,t+1}^m \le k_t^\star < k_{j,t+1}^M$ evade taxes and are caught in the dimensional trap.

Finally, consider Proposition 2.5, condition 3. In this case $q^\star \in [0, q_l]$ and no firm finds it profitable to evade taxes, so they all choose to invest all profit to maximize the expected profit in the next period. This situation is illustrated for Scenario 1, in Fig. 5, where the choice of the policy parameters leads to $q^\star = 0.638$, so $q^\star \in [0, q_l]$.

The illustrated findings are quite robust to changes in q_l, q_h, and m_0, provided that the relationship between q_l, q_h, and $q^\star$ is maintained. For example, using $q_l = 0.03$ and $q_h = 0.14$ – respectively, the probability of inspection for small and medium firms in Italy – the results are the same as in Fig. 3, for any value of m_0. Even choosing $m_0 = m_0^M$ gives $q^\star > q_h$. Similarly, letting $q_l = 0.14$ and $q_h = 0.32$, where the last value is the probability of inspection for large firms in Italy, results in the same evolutionary patterns as in Fig. 3, for $m_0 < 0.38$ (which gives $q^\star > q_h$),

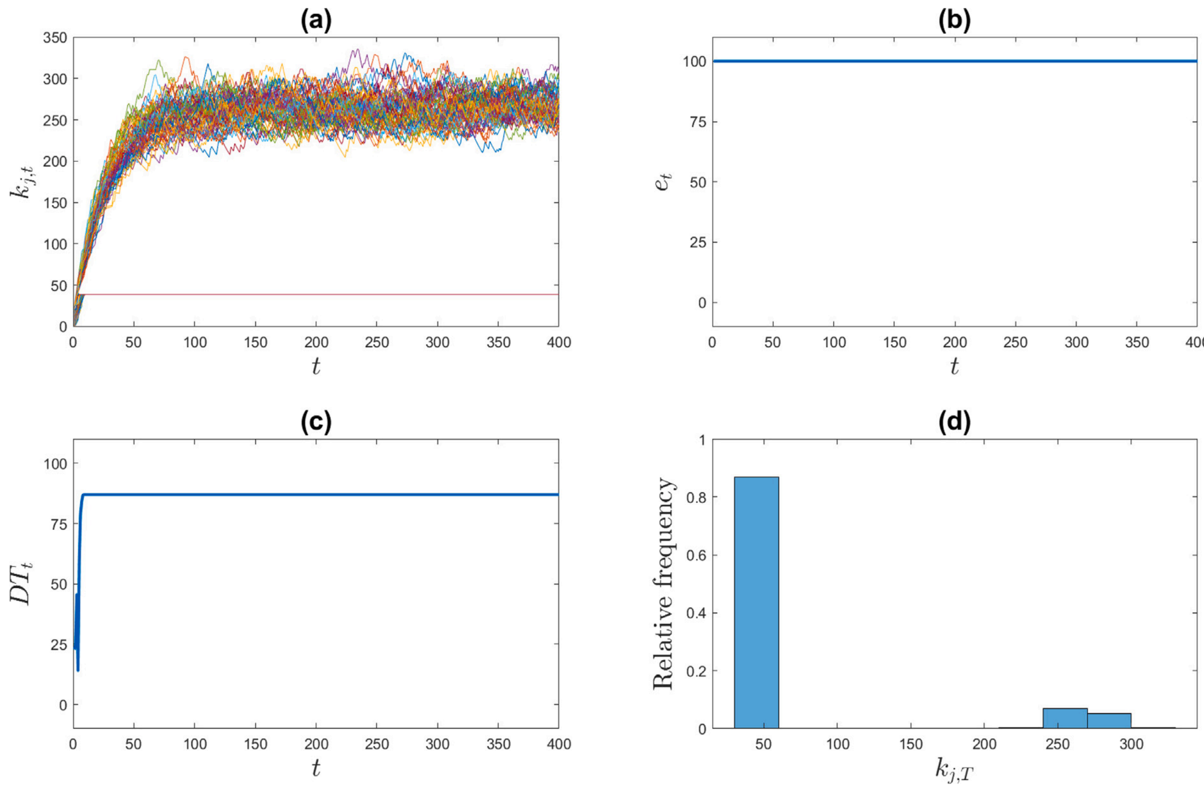

Fig. 3. Scenario 1. Temporal evolution of capital-per-capita level (a), evasion index (b), dimensional trap index (c), and distribution of the capital-per-capita level attained at time T (d). Policy parameters: $m_0 = 0.1$, $q_l = 0.2$, $q_h = 0.5$.

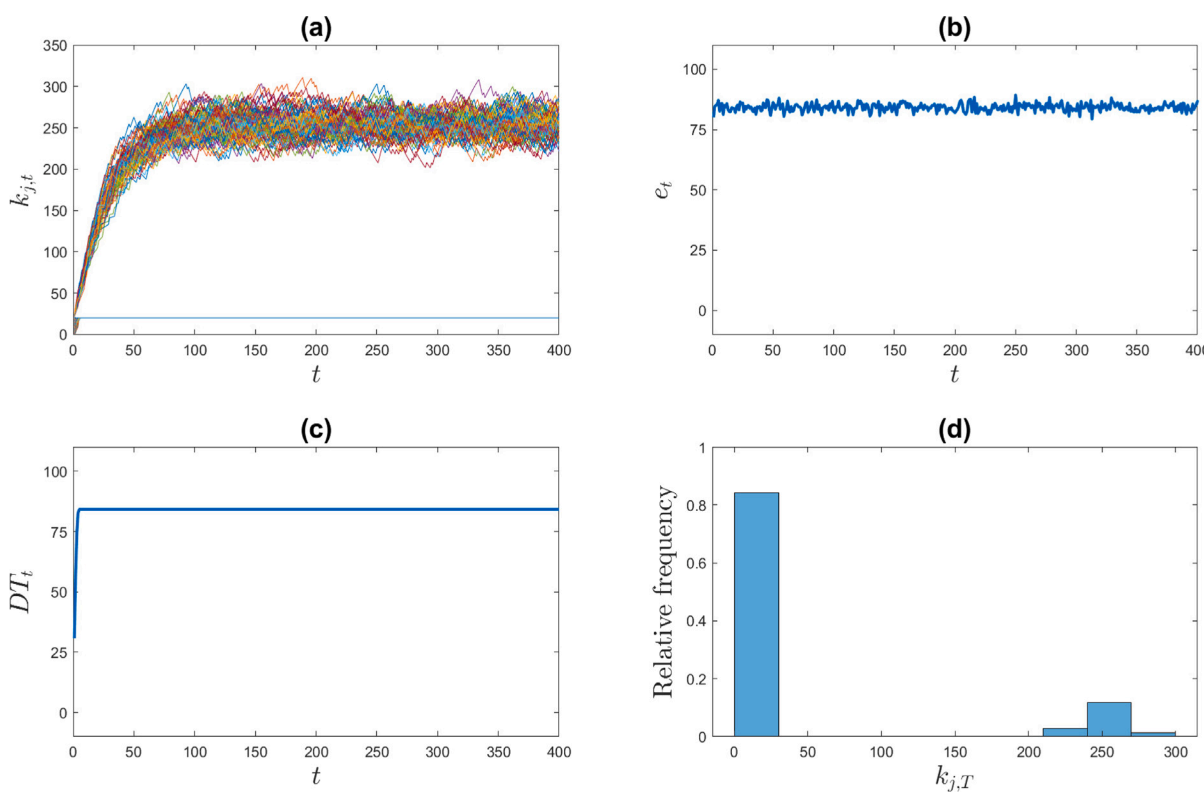

Fig. 4. Scenario 1. Temporal evolution of capital-per-capita level (a), evasion index (b), dimensional trap index (c), and distribution of the capital-per-capita level attained at time T (d). Policy parameters: $m_0 = 0.1$, $q_l = 0.2$, $q_h = 0.7$.

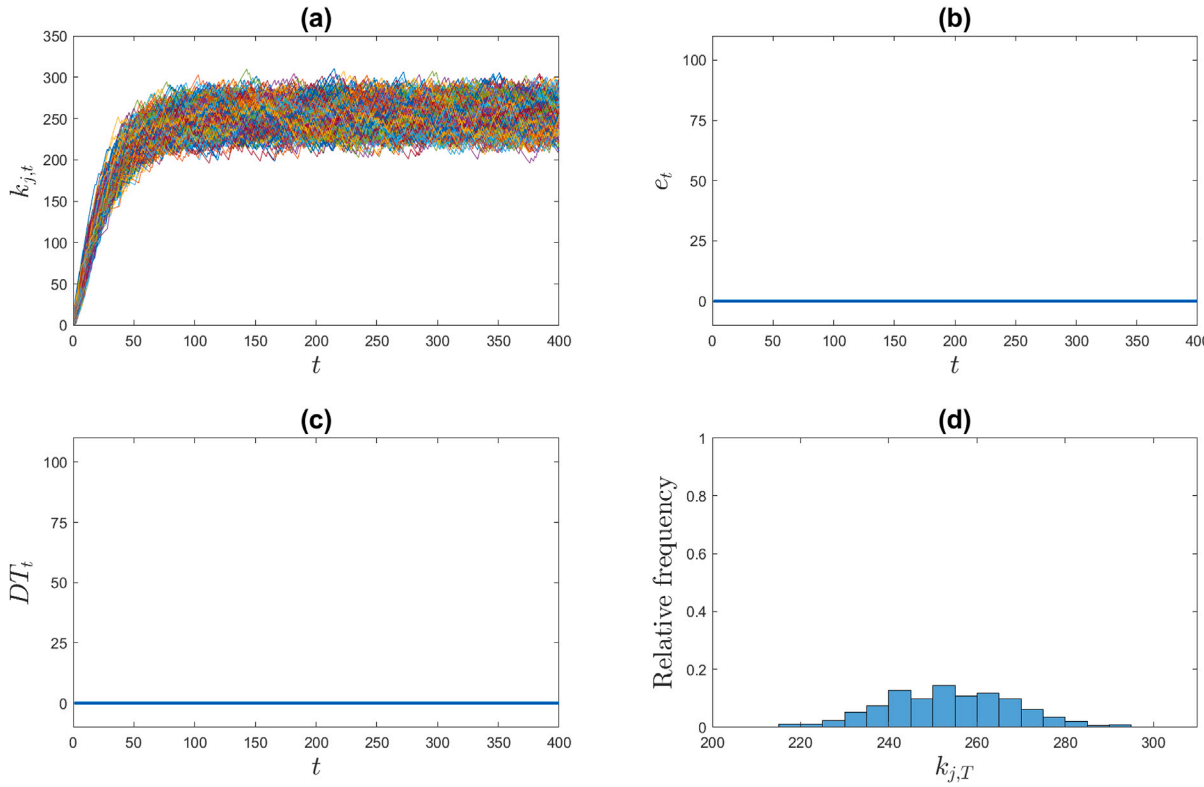

Fig. 5. Scenario 1. Temporal evolution of capital-per-capita level (a), evasion index (b), dimensional trap index (c), and distribution of the capital-per-capita level attained at time T (d). Policy parameters: $m_0 = 0.1$, $q_l = 0.7$, $q_h = 0.9$.

and the same as in Fig. 4 for $m_0 \geq 0.38$ (which gives $q_l < q^\star \leq q_h$). In either case, no value for m_0 can produce full compliance and growth.

While Figs. 3 to 5 explore the evolution of the capital-per-capita level, evasion, and dimensional trap indexes for a given value of m_0 and varying values of q_l and q_h, it is also interesting to investigate the long-term behavior of the industrial market for various combinations of fines and controls. For this purpose, we consider a grid of equally spaced values for m_0, from 0 to m_0^M, with a spacing of 0.02, and increasing values for q_h, namely, 0.4, 0.6, 0.8 and 1, while letting $q_l = 0.2$. For each combination of m_0 and q_h, we replicate the simulation 20 times for the temporal trajectories of the N firms. We then obtain at time T, for each combination and replication, the mean capital-per-capita level of firms caught in the dimensional trap, the mean capital-per-capita level of firms not caught in the dimensional trap, the evasion and the dimensional trap indexes. Finally, for each combination of policy parameters, the values are averaged over the 20 replications to mitigate possible sampling effects. The results for Scenario 1 are depicted in Fig. 6 together with the representation of the threshold for $k_t^\star$ (panel (d)), as defined in Eqs. (9) and (10). Starting from this last panel, note that the threshold for $k_t^\star$, above which staying small is profitable for at least some firms, is decreasing with m_0 and q_h, for m_0 such that $q^\star > q_h$ (see Eq. (9)), whereas for m_0 such that $q_l < q^\star \leq q_h$, the threshold increases with m_0 and depends only on this policy parameter[8] (see Eq. (10)). This has important consequences for the long-term growth of firms experiencing the dimensional trap, as their average capital-per-capita level settles slightly above this threshold value (with the distance from the threshold varying mildly from one simulation to another). This result is highlighted in the bottom left corner of panel (a). In addition, long-term growth for firms not experiencing the dimensional trap is affected by the level of m_0, provided that $q^\star > q_h$. In this situation, all firms find it convenient to evade (top left corner of panel (b)), and when their evasion is detected, their profit is reduced commensurate with the level of the fine (top left corner of panel (a)). Obviously, for a given m_0, the average capital-per-capita level of these firms decreases as q_h increases because the probability of evasion being detected is higher. For m_0 such that $q_l < q^\star \leq q_h$, evasion is no more convenient for large firms, as illustrated by the downward movement of the yellow, red, and blue lines in the top left part of panel (b), and they find it profitable to invest all profits. Consequently, the average level of capital-per-capita for these nontrapped firms no longer depends on q_h, and for a not-too-large m_0, it stays constant near its equilibrium level. If m_0 is further augmented, trapped firms that are detected for evasion receive such a high fine that their profit in that period, even if completely reinvested, is unable to offset the depreciation of their capital. Therefore, these firms can freely invest all their profits, as their capital-per-capita level will still be lower than $k_t^\star$, and in that period, they are no longer trapped. Their capital thus lowers the average capital-per-capita level of nontrapped firms, as shown in panel (a), where all the dashed lines start to drop for a sufficiently large m_0. This value of m_0 can be computed by solving:

$$(1-\delta)k_t^\star + (1-\tau)A_h(k_t^\star)^\alpha - m_0(A_h(k_t^\star)^\alpha - \tau A_l(k_t^\star)^\alpha) < k_t^\star \tag{11}$$

after substituting the lower bound in Eq. (10) into $k_t^\star$. Above this m_0 value, the average capital-per-capita of nontrapped firms continues to

[8] The smallest m_0 for which the different curves, corresponding to the different values of q_h, become coincident with the violet one (the one for $q_h = 1$), or, in other words, the value of m_0 for which each curve reaches its minimum, is the m_0 value for which $q^\star = q_h$.

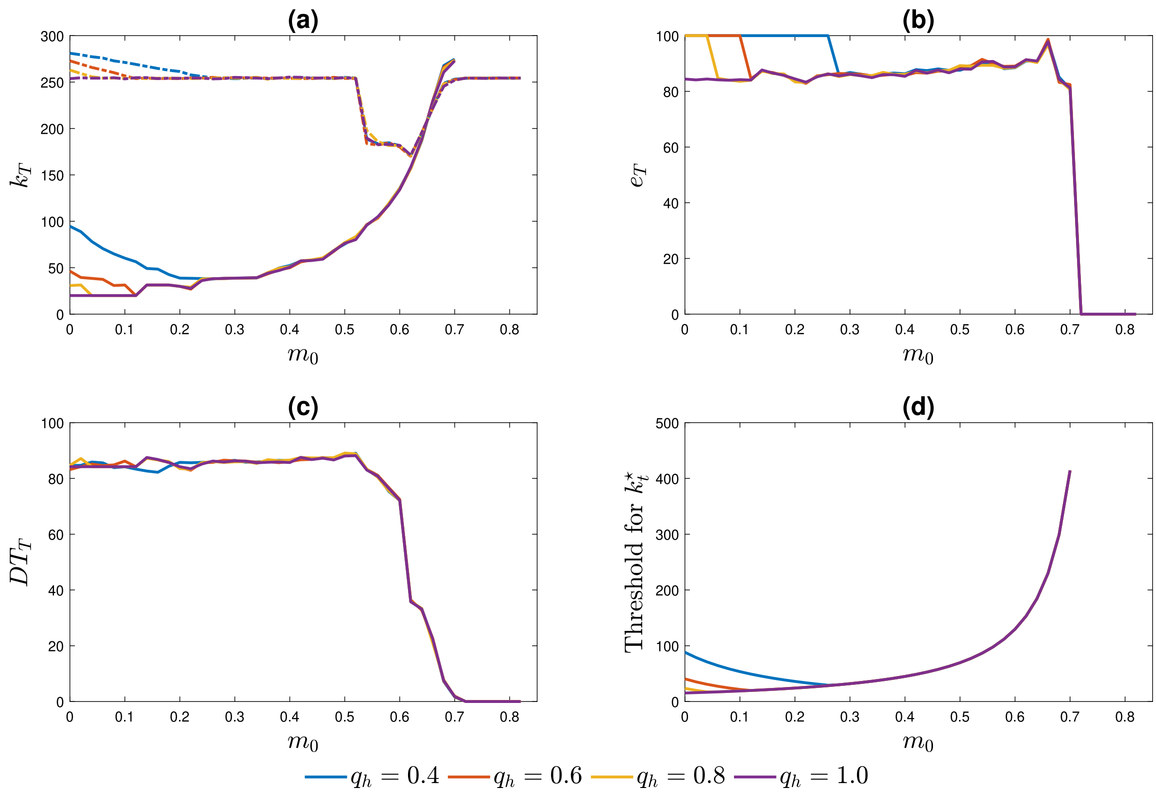

Fig. 6. Scenario 1. Long-term equilibrium, at time T, of different quantities of interest, for $q_l = 0.2$, different levels of q_h and increasing values of m_0: (a) mean capital-per-capita level of firms caught in the dimensional trap (continuous lines) and firms not caught in the dimensional trap (dashed lines), (b) evasion index, (c) dimensional trap index, (d) threshold for $k_t^\star$, as defined in (9) and (10).

decrease as m_0 increases until it equals that of trapped firms and starts to increase with m_0. As m_0 becomes sufficiently large that $q^\star \leq q_l$, evasion is no longer convenient for any firm, and the evasion index drops to zero (panel (b), bottom right corner), as does the dimensional trap index (panel (c), bottom right corner). Therefore, no firms are trapped, and their average capital-per-capita level equals the equilibrium value without evasion.[9]

Notably, for m_0 such that $q_l < q^\star \leq q_h$, the evasion index slowly increases with m_0. This is due to the increment of the threshold in Eq. (10), which makes it possible for a larger number of firms to stay small and find evasion convenient. A similar pattern can be seen for the dimensional trap index until m_0 is smaller than the threshold obtained from Eq. (10). Beyond this value, as indicated earlier, firms caught evading make such a small profit that even investing it all in the next period allows them to obtain only a capital-per-capita level smaller than $k_t^\star$ (that is, they fall into the situation where $k_{j,t+1}^M < k_t^\star$). Therefore, they evade without being in a dimensional trap (for a graphical representation, see the temporal patterns presented in Appendix A, Fig. A.1). Even if they invest all their profit, however, once their $k_{j,t}$ falls below $k_t^\star$, it will remain there in the future, constraining their growth.

In Fig. 6 the value of q_l is kept fixed at 0.2. Letting this policy parameter increase has the effect of shifting the threshold for $k_t^\star$ upward and decreasing the value of m_0 for which $q^\star \leq q_l$ and full tax compliance and growth are attained. These effects are graphically represented by Figs. B.1 and B.2 in Appendix B. Instead, letting γ increase pushes downward the dimensional trap index and, for $q_l < q^\star \leq q_h$, also the evasion index, as shown in Figs. C.1 and C.2 in Appendix C.

Summarizing, we can say that q_h is not particularly relevant as a policy parameter. In fact, if $q^\star \leq q_l$, the system experiences neither a dimensional trap nor evasion, no matter the value of q_h. Similarly, for $q_l < q^\star \leq q_h$, increasing or decreasing q_h (provided that it becomes no smaller than $q^\star$) does not modify the average growth of firms, the dimensional trap index, or the evasion index. Therefore, q_h levels have an effect only if $q^\star > q_h$, with smaller values of q_h promoting the growth of both trapped and nontrapped firms but also encouraging tax evasion. Instead, parameter q_l proves more effective. Unless $q^\star > q_h$, sufficiently increasing q_l leads to tax compliance and full growth of all firms. The most effective tool that the state has at its disposal to combat tax evasion and promote growth, is, though, the parameter m_0. Provided the value of q_l is not too small, one can always identify a sufficiently large value of m_0 to obtain $q^\star \leq q_l$ and thus perfect compliance and long-term firm growth. In particular, using Eq. (3), it is easy to show that the smallest value of m_0 for which these desirable conditions are attained is $m_0 = (1 - q_l)\tau(A_h - A_l)/[q_l(A_h - \tau A_l)]$, for $q_l \geq \tau(A_h - A_l)/(A_h - \tau A_l)$. Finally, γ is also an interesting policy tool, as increasing its value reduces the percentage of trapped firms (unless the system is already in a no-evasion and no-trap condition), and for $q_l < q^\star \leq q_h$, the percentage of firms evading taxes.

[9] Note that, just before the m_0 value for which $q^\star \leq q_l$, the average capital-per-capita level of trapped firms appears to be slightly larger than that of nontrapped firms. This is due to the fact that the capital-per-capita level of trapped and nontrapped firms are so close that firms switch easily from one condition to the other, and trapped ones at time T manage to attain a larger average capital-per-capita level than nontrapped ones, by evading taxes.

All the results previously illustrated seem robust to different starting scenarios. Findings similar to those in Fig. 6 can also be obtained for systems initially characterized by firms that are all approximately the same size,[10] a small number of large firms, or a large number of small firms. For this reason, specific results are not presented here, but a graphical summary is provided in Appendix D, Figs. D.1 to D.3.

4. Discussion, conclusions, and further developments

This paper considers size-dependent tax enforcement strategies and models their effects on firms' tax compliance, investment decisions, and global industrial structure. In particular, we endogenized the audit probability with respect to firms' size, assuming two different levels of control, respectively, for small and large firms. We then developed a dynamic model to evaluate the effects of this size-dependent regulation. Combining analytical and simulation results, our work showed that, depending on policy parameter values, size-dependent enforcement may fail in contrasting evasion and have the unintended effect of encouraging firms to remain small to avoid a higher level of enforcement, a situation we define as a dimensional trap. The ultimate result, in the long run, could be the establishment of an industrial structure made up of a few large firms and a plethora of small firms, with the latter characterized by inefficient allocation of resources and noncompliant behavior with fiscal regulations.

We focused specifically on the particular combinations of policy parameters that can determine the spread of tax evasion and the arising of the dimensional trap. We mainly considered the monitoring probabilities of small and large firms, q_l and q_h, respectively, and the fine, m_0, prescribed for tax evaders. This last parameter is crucial, as it is inversely related to the monitoring threshold $q^\star$, which determines whether the monitoring level faced by each firm is small enough for evasion to be profitable. The results show that a dimensional trap can arise for small firms if $q_h < q^\star$ (in combination with evasion of both small and large firms), or $q_l < q^\star \leq q_h$ (in combination with evasion of small firms only), while $q_l \geq q^\star$ necessarily implies full compliance and growth. These outcomes are robust to changes in q_l, q_h and m_0, provided that the relationship between q_l, q_h and $q^\star$ is maintained. In addition, different starting industrial configurations, for given values of the policy parameters, seem to converge to analogous structures in the long run, with similar percentages of evading and trapped firms.

The findings of our work are particularly interesting when considering enforcement in Italy, where small and medium firms face respective monitoring probabilities of $q_l = 0.03$ and $q_h = 0.14$. According to our model, these values of q_l and q_h are so small that $q_h < q^\star$, for any possible value of the fine, indicating evasion by both small and medium firms and a dimensional trap for small ones. When the respective audit probabilities for medium and large firms in Italy are compared – $q_l = 0.14$ and $q_h = 0.32$, respectively – we obtain $q^\star > q_h$ for $m_0 < 0.38$ (i.e., evasion by both medium and large firms and a dimensional trap for medium ones) and $q_l < q^\star \leq q_h$ for $m_0 \geq 0.38$ (i.e., evasion by and a dimensional trap for medium firms). In either case, no value for the fine m_0 can produce full compliance and growth. In summary, our model suggests that the audit probabilities of small, medium, and large firms in Italy are so low that they favor the spread of evasion and encourage firms to constrain their growth to avoid higher levels of control. A similar situation can be found, for example, in the U.S., where Martin et al. (2020) highlight that the probability of firms auditing has declined steadily in the last thirty years, moving, for large firms, from a value of 0.56 in 1991 to a value of 0.14 in 2017 (see Table 1 in Martin et al., 2020). According to our model, this last value is so small that even very high fine levels would not be able to contrast evasion. In addition, in 2017, the audit probabilities across firms of different size classes are such as to favor the emergence of dimensional traps more than those registered in 1991. This might contribute to explain the 25% increase in concentration among U.S. firms since the mid-1990s.

For Spain, Almunia and Lopez-Rodriguez (2018) provide evidence that the establishment of LTUs causes firms to bunch below the eligibility threshold to avoid stricter tax enforcement. Audit probabilities for small and large firms are not provided, however, as the LTU has more auditors per taxpayer than the rest of the tax authority, and those auditors have on average higher qualifications and experience, we can guess that the gap between q_l and q_h is large enough for our model to envisage a situation of dimensional trap, following the findings of Almunia and Lopez-Rodriguez (2018).

The case of Germany is, instead different. Whereas size-dependent audit probabilities in this country are compatible,[11] according to our results, with a situation of dimensional trap for medium, small, and very small firms, Klimsa and Ullmann (2023) find no strategical bunching of firms below the size thresholds. Even this evidence, however, finds an explanation under our approach. The specific design of the audit target selection process (see Klimsa and Ullmann, 2023, for further details), is such that audit probability positively correlates with firm size within size classes, thus attenuating the discontinuities in audit probability at the size thresholds. Such a reduction in the distance between q_l and q_h hinders the emergence of the dimensional trap in our framework. An analogous argument applies to the case of Jamaica, where size thresholds are based on a combination of taxes paid and revenue, and for which Tennant and Tracey (2019) conclude that a more multilayered threshold-dependent policy improves firms' tax compliance without causing tax-induced size management.

To conclude, we believe that our work can provide some interesting policy guidelines. For example, the probability of being checked for a bigger firm, q_h, does not play a particularly significant role as a policy parameter. In fact, if $q^\star \leq q_l$, the system experiences neither dimensional trap nor evasion, no matter the value of q_h. Similarly, for $q_l < q^\star \leq q_h$, increasing or decreasing q_h (provided that it becomes no smaller than $q^\star$) does not modify the average growth of firms, the percentage of those trapped, and the percentage of those evading taxes. The probability of being checked for smaller firms, q_l, demonstrates, instead, greater effectiveness. Unless $q^\star > q_h$, sufficiently increasing q_l leads to tax compliance and full growth of all firms. The state's most powerful instrument for tackling tax evasion and fostering growth, is, though, the fine, m_0, envisaged for the tax evaders. For not-too-small values of q_l, one can always identify a sufficiently large value of m_0 to obtain $q^\star \leq q_l$ and, thus, perfect compliance and long-term firm growth. However, when tax enforcement is inefficient, increasing the cost of tax evasion for firms does not reduce tax evasion and might trigger perverse side effects (see also Dzhumashev et al., 2023, on this point). Finally, the monitoring effort put in place by the state to reduce evasion (i.e., the proportion γ of firms that the state is willing to audit with a higher probability) is also an interesting policy tool, as increasing

[10] As described previously, Scenario 2 is obtained by setting the value of $k_{j,0}$ equal to 10 plus a small perturbation obtained by drawing random values from a standard normal distribution. Some experiments have also been conducted using values drawn from a normal distribution with a larger variance in order to obtain initial values of the capital-per-capita distributed in a bell-shaped manner with greater dispersion. However, even in this case, no substantial changes in the results have been observed.

[11] As described in Klimsa and Ullmann (2023), for the year 2010, 21.1% of large firms were audited as opposed to only 6.9% of medium firms, 3.5% of small firms and 1% of very small firms.

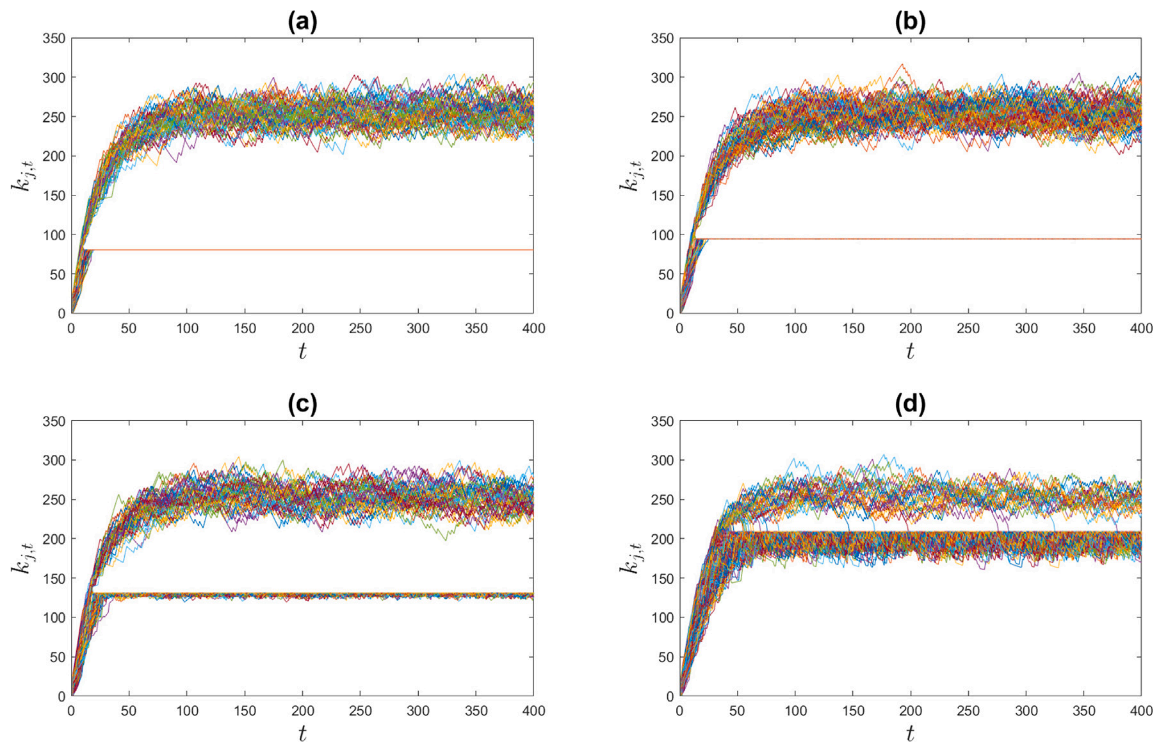

Fig. A.1. Temporal evolution of capital-per-capita level, for $q_l = 0.2$ and $q_h = 0.7$. For $m_0 = 0.5$, we have $e_T = 89.0\%$ and $DT_T = 89.0\%$, i.e., evading firms are also trapped (a); for $m_0 = 0.55$, we have $e_T = 89.96\%$ and $DT_T = 79.6\%$, i.e., evading firms that are inspected realize such a small profit that in the next period they experience $k^M_{j,t} < k^\star_t$ and they are not trapped (b); for $m_0 = 0.6$, we have $e_T = 90.95\%$ and $DT_T = 71.2\%$, similarly to previous case (c); for $m_0 = 0.65$, we have $e_T = 93.49\%$ and $DT_T = 31.6\%$, i.e., even large firms can experience a $k_{j,t} < k^\star_t$ at some point, if they face a bad state of nature more than once in a row, and afterwards find it worthwhile to evade taxes (d).

its value reduces the percentage of trapped firms (unless the system is already in a no-evasion and no-trap condition), and for $q_l < q^\star \le q_h$, the percentage of firms evading taxes.

In future work, the framework can be extended and refined in different ways. For example, by considering a general equilibrium model in which the monitoring effort, γ, the state puts in fighting evasion is endogenous and depends on taxes collected by the state. In this case, γ can be assumed to be a continuous and strictly increasing function of the tax amount the state can gain from firms' taxation. Another possibility is to explore various functional production forms and costs to assess the robustness of the outcomes. Ultimately, it might be interesting to create a model to estimate the likelihood of experiencing a favorable state of nature, taking into consideration economic cycles and variations among firms.

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Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Matlab routines used for simulating the dynamic model presented in the paper are available at the following link: https://data.mendeley.com/datasets/spjgxmjzbb/1.

Appendix A. Temporal patterns for increasing values of m_0 within $q_l < q^\star \le q_h$

See Fig. A.1.

Appendix B. Effects of increasing q_l

See Figs. B.1 and B.2.

Appendix C. Effects of increasing γ

See Figs. C.1 and C.2.

Appendix D. Long term equilibrium under different starting scenarios

See Figs. D.1–D.3.

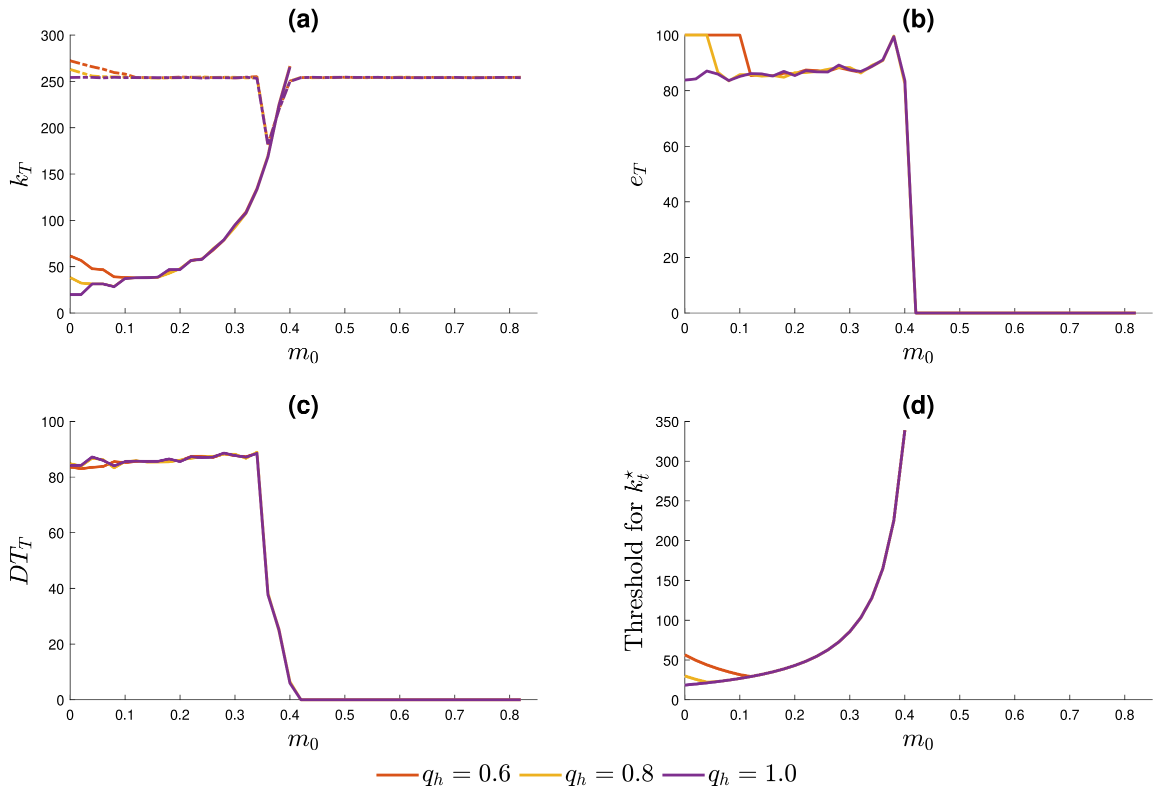

Fig. B.1. Long term equilibrium, at time T, of different quantities of interest, for $q_l = 0.3$, different levels of q_h and increasing values of m_0: (a) mean capital-per-capita level of firms caught in the dimensional trap (continuous lines) and firms not caught in the dimensional trap (dashed lines), (b) evasion index, (c) dimensional trap index, (d) threshold for $k_t^\star$, as defined in (9) and (10).

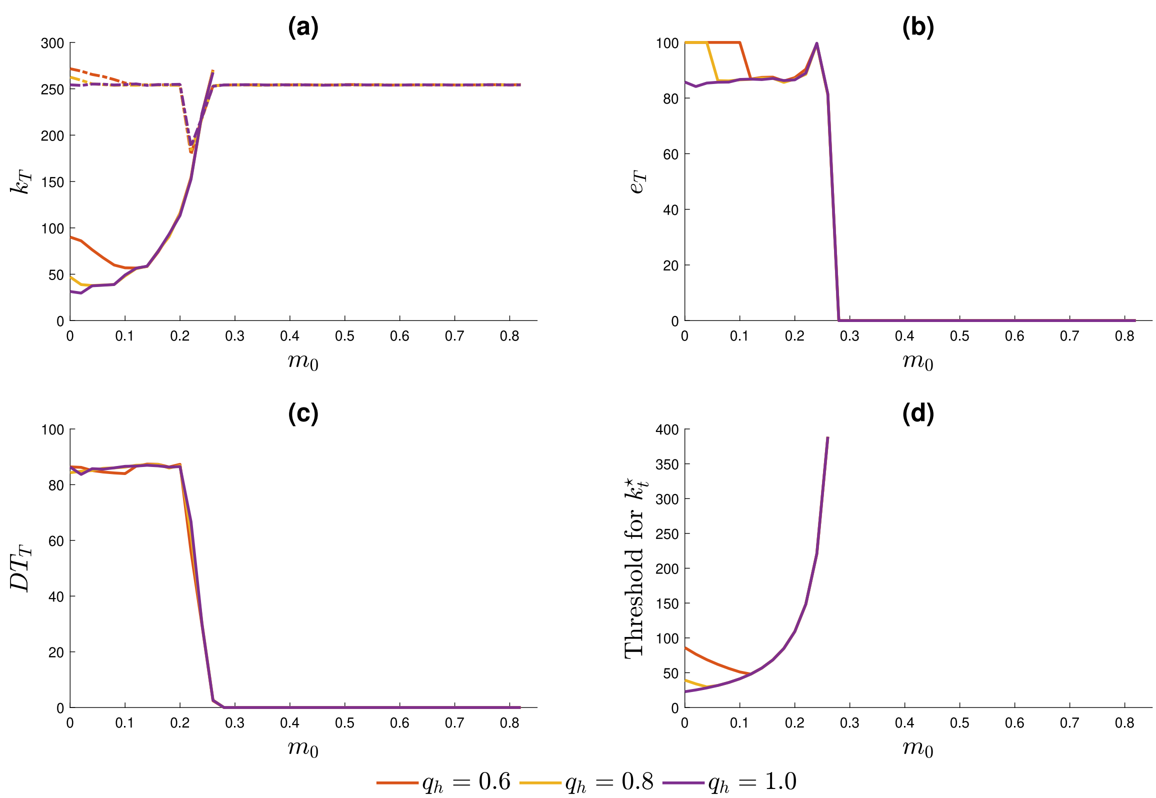

Fig. B.2. Long term equilibrium, at time T, of different quantities of interest, for $q_l = 0.4$, different levels of q_h and increasing values of m_0: (a) mean capital-per-capita level of firms caught in the dimensional trap (continuous lines) and firms not caught in the dimensional trap (dashed lines), (b) evasion index, (c) dimensional trap index, (d) threshold for $k_t^\star$, as defined in (9) and (10).

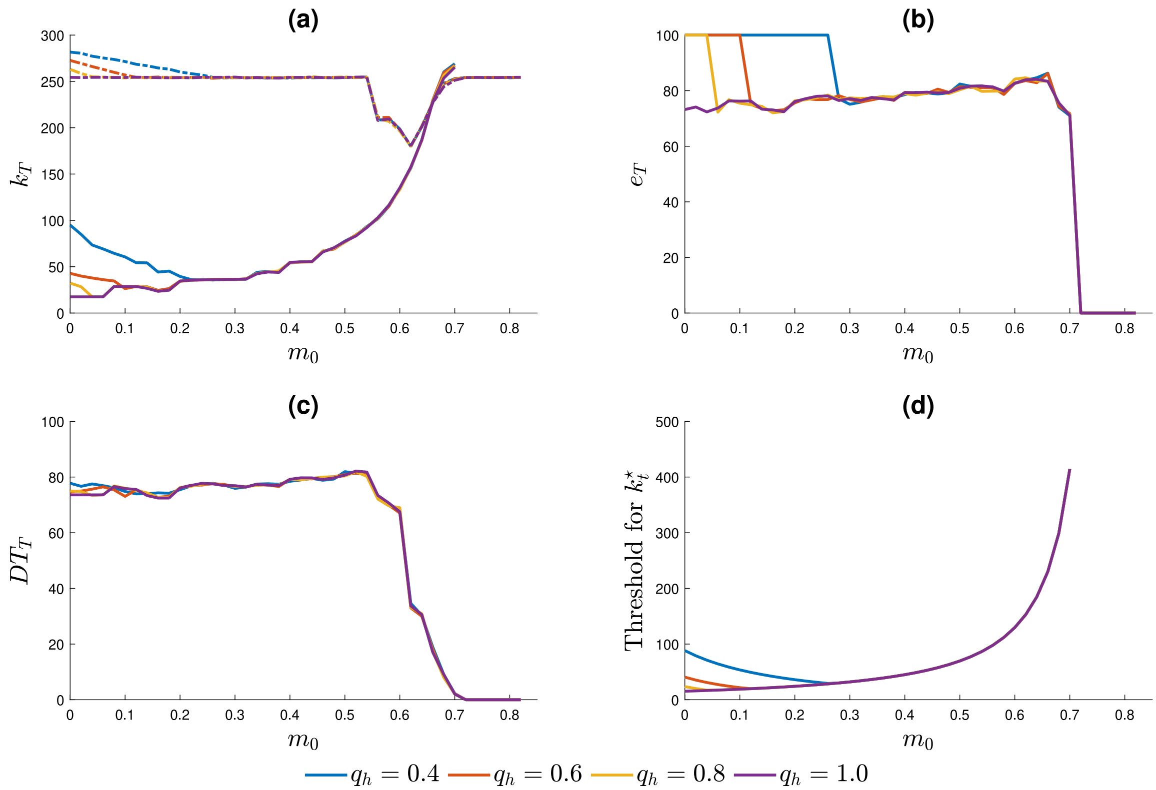

Fig. C.1. Long term equilibrium, at time T, of different quantities of interest, for $q_l = 0.2$, different levels of q_h, increasing values of m_0, and $\gamma = 0.3$: (a) mean capital-per-capita level of firms caught in the dimensional trap (continuous lines) and firms not caught in the dimensional trap (dashed lines), (b) evasion index, (c) dimensional trap index, (d) threshold for $k_t^\star$, as defined in (9) and (10).

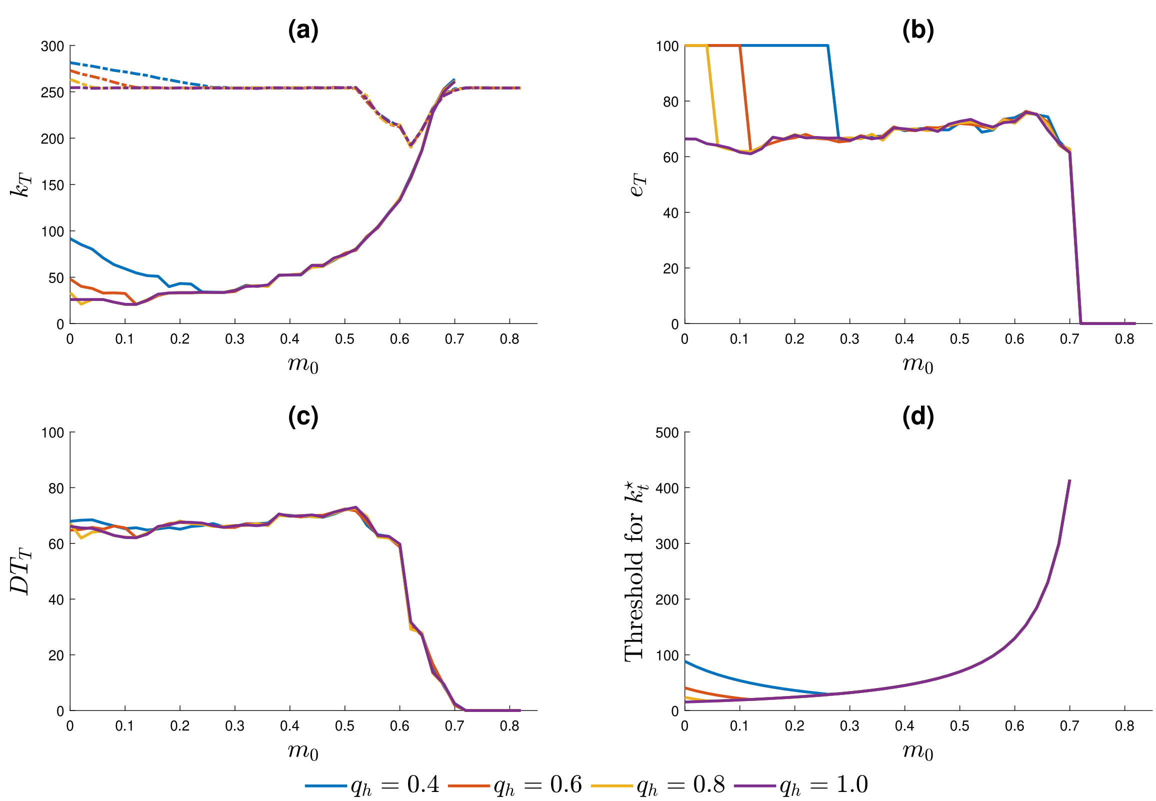

Fig. C.2. Long term equilibrium, at time T, of different quantities of interest, for $q_l = 0.2$, different levels of q_h, increasing values of m_0, and $\gamma = 0.4$: (a) mean capital-per-capita level of firms caught in the dimensional trap (continuous lines) and firms not caught in the dimensional trap (dashed lines), (b) evasion index, (c) dimensional trap index, (d) threshold for $k_t^\star$, as defined in (9) and (10).

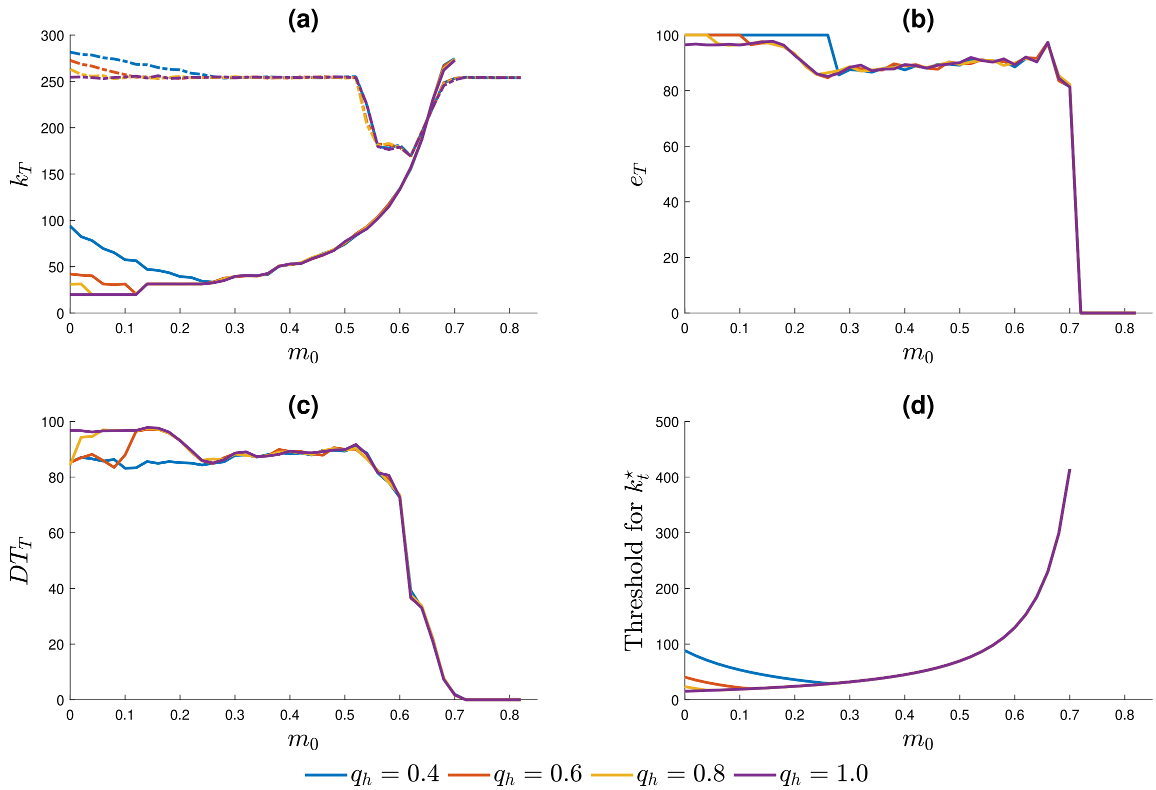

Fig. D.1. Scenario 2. Long term equilibrium, at time T, of different quantities of interest, for $q_l = 0.2$, different levels of q_h and increasing values of m_0: (a) mean capital-per-capita level of firms caught in the dimensional trap (continuous lines) and firms not caught in the dimensional trap (dashed lines), (b) evasion index, (c) dimensional trap index, (d) threshold for $k_t^\star$, as defined in (9) and (10).

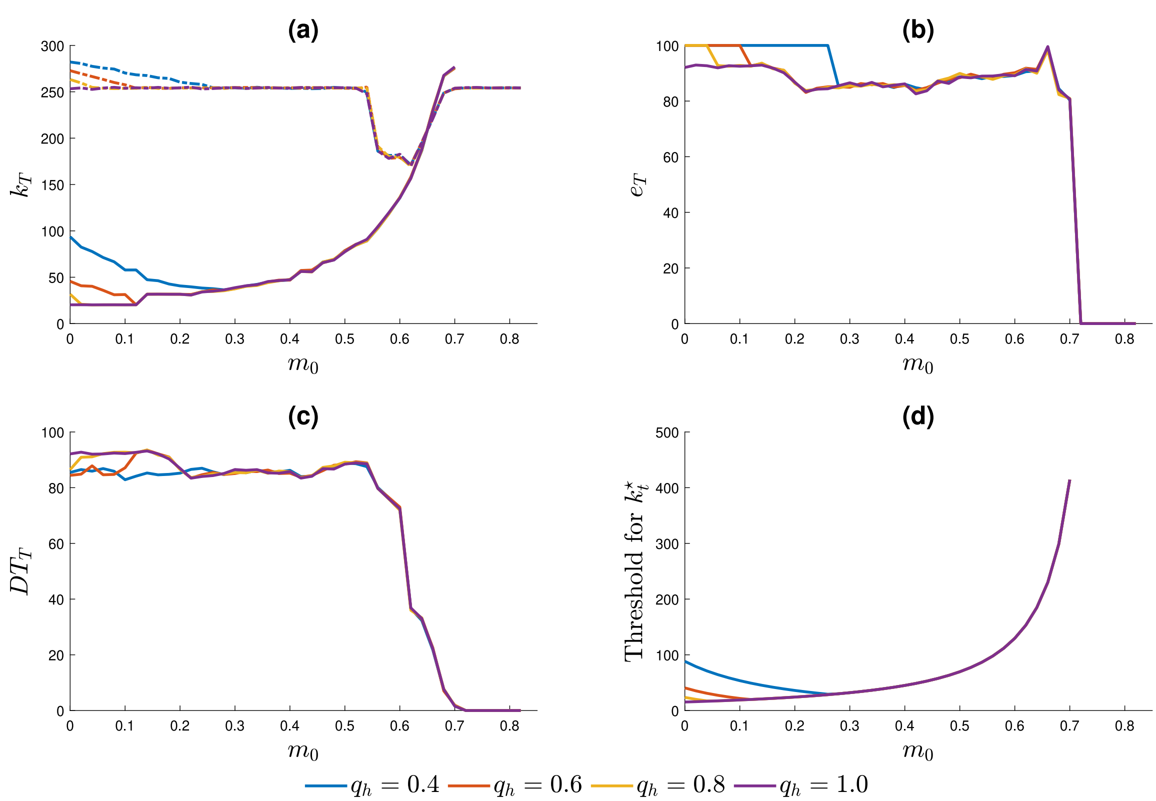

Fig. D.2. Scenario 3. Long term equilibrium, at time T, of different quantities of interest, for $q_l = 0.2$, different levels of q_h and increasing values of m_0: (a) mean capital-per-capita level of firms caught in the dimensional trap (continuous lines) and firms not caught in the dimensional trap (dashed lines), (b) evasion index, (c) dimensional trap index, (d) threshold for $k_t^\star$, as defined in (9) and (10).

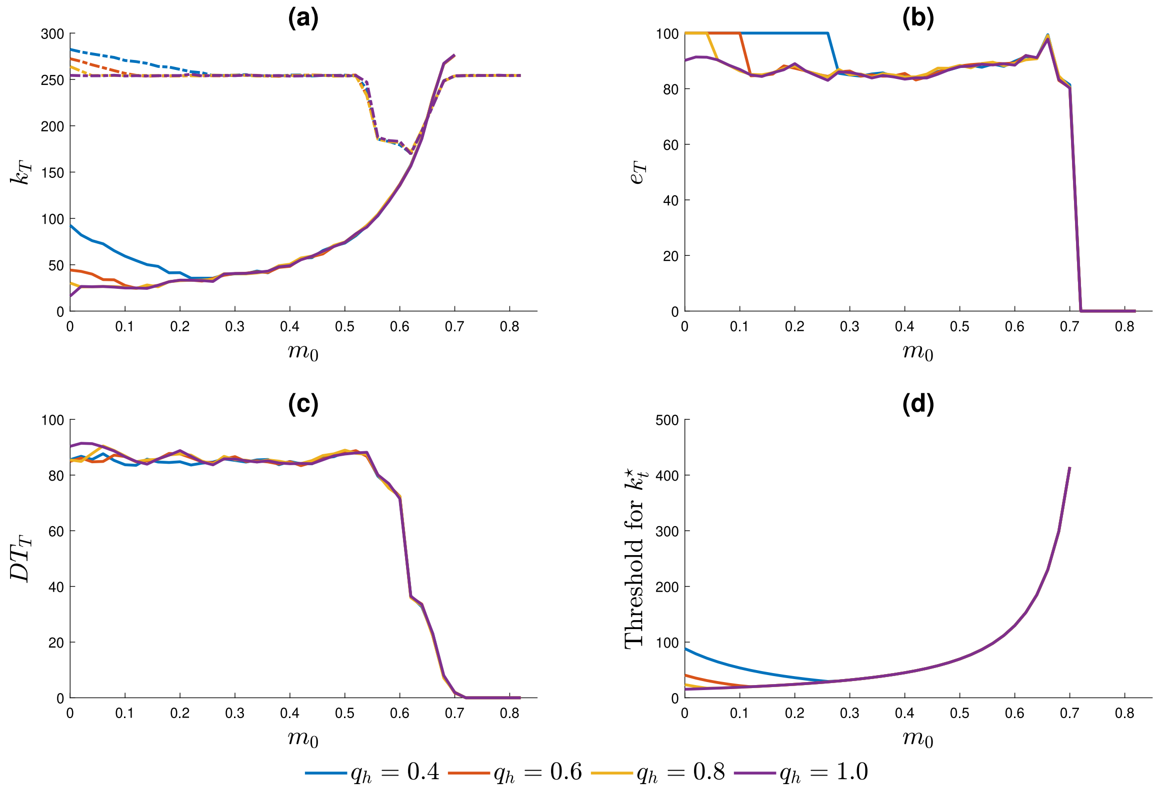

Fig. D.3. Scenario 4. Long term equilibrium, at time T, of different quantities of interest, for $q_l = 0.2$, different levels of q_h and increasing values of m_0: (a) mean capital-per-capita level of firms caught in the dimensional trap (continuous lines) and firms not caught in the dimensional trap (dashed lines), (b) evasion index, (c) dimensional trap index, (d) threshold for $k_t^\star$, as defined in (9) and (10).

References

Allingham, M.G., Sandmo, A., 1972. Income tax evasion: A theoretical analysis. J. Public Econ. 1 (3–4), 323—338. http://dx.doi.org/10.1016/0047-2727(72)90010-2.

Almunia, M., Lopez-Rodriguez, D., 2018. Under the radar: The effects of monitoring firms on tax compliance. Am. Econ. J.: Econ. Policy 10 (1), 1—38. http://dx.doi.org/10.1257/pol.20160229.

Bachas, P., Jaef, R.N.F., Jensen, A., 2019. Size-dependent tax enforcement and compliance: Global evidence and aggregate implications. J. Dev. Econ. 140, 203—222. http://dx.doi.org/10.1016/j.jdeveco.2019.05.001.

Bethencourt, C., Kunze, L., 2020. Social norms and economic growth in a model with labor and capital income tax evasion. Econ. Model. 86, 170—182. http://dx.doi.org/10.1016/j.econmod.2019.06.009.

Boonzaaier, W., Harju, J., Matikka, T., Pirttilä, J., 2019. How do small firms respond to tax schedule discontinuities? Evidence from South African tax registers. Int. Tax Public Finance 26 (5), 1104—-1136. http://dx.doi.org/10.1007/s10797-019-09550-z.

Buehn, A., Schneider, F.G., 2012. Size and development of tax evasion in 38 OECD countries: What do we (not) know? SSRN Electron. J. 4004, http://dx.doi.org/10.2139/ssrn.2181359.

Cerqueti, R., Coppier, R., 2011. Economic growth, corruption and tax evasion. Econ. Model. 28 (1–2), 489—500. http://dx.doi.org/10.1016/j.econmod.2010.07.006.

CGIA-MESTRE, 2019. Fisco: Le Grandi Imprese Evadono 16 Volte in Piú delle Piccole. https://www.cgiamestre.com/wp-content/uploads/2019/10/Accertamenti-fiscali-05.10.2019.pdf.

Coppier, R., Michetti, E., 2006. Corruption vs production. A non-linear relationship. Econ. Model. 23 (4), 622—637. http://dx.doi.org/10.1016/j.econmod.2006.03.003.

Coppier, R., Michetti, E., Scaccia, L., 2022. Industrial structure and evasion dynamics, is there any link? Metroeconomica 73 (4), 960—986. http://dx.doi.org/10.1111/meca.12364.

Coppier, R., Michetti, E., Scaccia, L., 2023. Size-dependent enforcement, tax evasion and dimensional trap. Comput. Econ. http://dx.doi.org/10.1007/s10614-023-10399-y.

Dzhumashev, R., Levaggi, R., Menoncin, F., 2023. Optimal tax enforcement with productive public inputs. Econ. Model. 126, 106444. http://dx.doi.org/10.1016/j.econmod.2023.106444.

Fernández-Bastidas, R., 2023. Entrepreneurship and tax evasion. Econ. Model. 128, 106488. http://dx.doi.org/10.1016/j.econmod.2023.106488.

Gourio, F., Roys, N., 2014. Size-dependent regulations, firm size distribution, and reallocation. Quant. Econ. 5 (2), 377—416. http://dx.doi.org/10.3982/QE338.

Guner, N., Ventura, G., Xu, Y., 2008. Macroeconomic implications of size-dependent policies. Rev. Econ. Dyn. 11 (4), 721—744. http://dx.doi.org/10.1016/j.red.2008.01.005.

Klimsa, D., Ullmann, R., 2023. Threshold-dependent tax enforcement and the size distribution of firms: Evidence from Germany. Int. Tax Public Finance 30 (4), 1002—1035. http://dx.doi.org/10.1007/s10797-022-09732-2.

López, J.J., Torres, J., 2020. Size-dependent policies, talent misallocation, and the return to skill. Rev. Econ. Dyn. 38, 59—93. http://dx.doi.org/10.1016/j.red.2020.03.005.

Luttmer, E.F.P., Singhal, M., 2014. Tax morale. J. Econ. Perspect. 28 (4), 149—168. http://dx.doi.org/10.1257/jep.28.4.149.

Martin, J., Parenti, M., Toubal, F., 2020. Corporate tax avoidance and industry concentration. SSRN Electron. J. 8469, http://dx.doi.org/10.2139/ssrn.3671255.

Murphy, R., 2019. The European Tax Gap - A Report for the Socialists and Democrats Group in the European Parliament. Global Policy. https://www.socialistsanddemocrats.eu/it/publications/european-tax-gap.

Onji, K., 2009. The response of firms to eligibility thresholds: Evidence from the Japanese value-added tax. J. Public Econ. 93 (5–6), 766—775. http://dx.doi.org/10.1016/j.jpubeco.2008.12.003.

Poniatowski, G., Bonch-Osmolovskiy, M., Śmietanka, A., Pechcińska, A., 2022. VAT Gap in the EU —- Report 2022. European Commission and Directorate-General for Taxation and Customs Union, Publications Office of the European Union, http://dx.doi.org/10.2778/109823.

Ramaswamy, K.V., 2021. Do size-dependent tax incentives discourage plant size expansion? evidence from panel data in Indian manufacturing. Margin: J. Appl. Econ. Res. 15 (4), 395—417. http://dx.doi.org/10.1177/09738010211036295.

Tennant, S.N., Tracey, M.R., 2019. Corporate profitability and effective tax rate: The enforcement effect of large taxpayer units. Account. Bus. Res. 49 (3), 342—361. http://dx.doi.org/10.1080/00014788.2018.1512398.