A Mixed Integer Linear Programming Approach to Markov Chain Bootstrapping

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Abstract

Bootstrapping time series is one of the most acknowledged tools to make forecasts and study the statistical properties of an evolutive phenomenon. The idea underlying this procedure is to replicate the phenomenon on the basis of an observed sample. One of the most important classes of bootstrap procedures is based on the assumption that the sampled phenomenon evolves according to a Markov chain. Such an assumption does not apply when the process takes values in a continuous set, as frequently happens for time series related to economic and financial variables. In this paper we apply Markov chain theory for bootstrapping continuous processes, relying on the idea of discretizing the support of the process and suggesting Markov chains of order k to model the evolution of the time series under study. The difficulty of this approach is that, even for small k, the number of rows of the transition probability matrix is too large, and this leads to a bootstrap procedure of high complexity. In many practical cases such complexity is not fully justified by the information really required to replicate a phenomenon satisfactorily. In this paper we propose a methodology to reduce the number of rows without loosing "too much" information on the process evolution. This requires a clustering of the rows that preserves as much as possible the "law" that originally generated the process. The novel aspect of our work is the use of Mixed Integer Linear Programming for formulating and solving the problem of clustering similar rows in the original transition probability matrix. Even if it is well known that this problem is computationally hard, in our application medium size real-life instances were solved efficiently. Our empirical analysis, which is done on two time series of prices from the German and the Spanish electricity markets, shows that the use of the aggregated transition probability matrix does not affect the bootstrapping procedure, since the characteristic features of the original series are maintained in the resampled ones.

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