Multiplier impact of wine activity on interindustry interactions

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Abstract

The increasing relevance of wine sector on the productive structure requires additional economic considerations on the economic and social impacts of the national and regional policies. Our work tries to analyze such policy impacts, by means of a multisectoral approach, in order to identify the strength of the links of the wine activity with all the other economic activities. Since wine is forwarded for a greater share to final demand, it is possible to determine the impacts of changes of demand in such activity on the whole economic system. Our analysis requires both the construction of an Input-Output table where wine is conveniently allocated, and its further extension in a context of Social Accounting Matrix, in order to evaluate the effects on the productive structure, of shocks, on primary and secondary distribution of income.

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1 Introduction

In last five years the production process of wine in Italy has undergone major changes. Though a lot of producers have remained small, output is obtained and marketed in a way much similar to manufacturing products. The Italian Statistical Office (ISTAT) in its last report gives the distribution shares of wine producers: only a 40-45 per cent is given by farmer.

Wine output is usually treated in agricultural economics and considered agricultural product. A high share of wine output is allocated to final demand. The role of wine within agricultural output has become increasingly relevant for its function of leading activity it has assumed in recent decades. This role is confirmed by recent trends in wine exports whose share on agriculture export has relevantly grown.

Inside the European Union Italian wine is present with about 322 wine Doc, 21 Docg and 113 Itg. The PAC (Communitarian Agrarian Policies) will have to assume an increasing relevance on the growth of this activity given that EU continues to increase the degree of openedness towards other European and extra-European countries. Subsidies can be assigned on the basis of either total output or value added and the results can be significantly different.

The analysis of the PAC for wine requires a clear picture of the inter industry relations at a high level of the detail. Moreover, if the Pac, affects income generation through direct transfer to producers incomes or subsides to investment, it is important to state formally the relationships between income by institutional sectors and output by industries.

In our analysis we attempt to model these links with the objective to give a picture of wine activities in national accounts. Hence we will need to build a multi sectoral and multi industry and a Social Accounting Matrix to provide a consistent data base to it.

In section 2 we note how recent developments in national accounting make implicit reference to an extended income output circular flow which integrates output implentation and income distribution. We show how, within this framework, we derived the loop disposable income/total output. In section 3 an explicit reference is made to the problem of isolating two wine sectors within a regional Input-Output table integrating data from different accounting sources. In section 4 a new method of impact analysis is presented that rests on consistent definition of macro multipliers. The proposed method combines spectral analysis with correlation analysis and produces a ranking of sectors and industries. Such ranking shows how the multipliers "ruling" the results perceive the change in each sector income and how industry outputs are affected by the set multipliers activated. A further cross correlation scrutiny of data gives the measure of relevance of how each industry output change has been affected by each change in sector income. The method has been applied with special reference to the special role of wine among industrial and sectoral interactions.

2 The multisectorial approach

Recent development in National Accounting have realized a substantial progress in the accounting system that integrates the keynesian income-expenditure model with the leontievian total output-intermediate consumption framework. In this way the emerging accounting scheme makes reference to an enlarged income circular flow. Final demands generate outputs and value added at industry level, which is distributed to factors and, through these, to institutional sectors in order to obtain, after taxation, disposable income by institutional sectors. These sectors will determine personal consumption expenditure and investment which will constitute final demand by industry.[1]. The complete model (Ciaschini,M. and Socci,C. 2002 [5]) allows for the reconstruction of the income loop from the output side to the income distribution side. In this paper we perform a partial analysis of the income circular flow concentrating only on the links between sectoral disposable incomes/ final demand and final demand /total output.

The direct and indirect output requirements for the final demand vector $\mathbf{f}$ is easily written in terms of the inverse

$$\mathbf{x} = [\mathbf{I} - \mathbf{A}]^{-1} \cdot \mathbf{f} \tag{1}$$

where $\mathbf{A}$ is the coefficients matrix usually determinate. As $M \cdot \widehat{\mathbf{x}}^{-1}$ and $\widehat{\mathbf{x}}$ is the diagonal matrix of total output.

The final demand formation (by IO sectors)

$$\mathbf{f} = [\mathbf{F}^0 + \mathbf{K}] \cdot \mathbf{y} + \mathbf{f}^0 \tag{2}$$

where $\mathbf{F}^0$ provides the consumption demand structure by industry and is given by the product of two matrices, $\mathbf{F}^0 = \mathbf{F}^1 \cdot \mathbf{C}$, where $\mathbf{F}^1$ [13,7] transforms the consumption expenditure by institutional sector into consumption by IO sectors and $\mathbf{C}$ [7,7] represents the consumption propensities by institutional sector.[2] $\mathbf{K}$ represents the investment demand and is given by $\mathbf{K} = \mathbf{K1} \cdot \mathbf{s} \cdot (\mathbf{I} - \mathbf{C})$ where $\mathbf{K1}$ [13,7] represents the investment demands to Input-Output sectors and $\mathbf{s}$ is a scalar that

[1] In the intersectoral table used for empirical analysis the producing activities are given by the branches of homogeneous production.

[2] To see the appendix for the Input-Output and Institutional Sector

represents the share of private savings which is transformed into investment i.e. "active savings"; vector $\mathbf{y}$ represents disposable income of the institutional sectors and in our application will be considered as exogenous. $\mathbf{f}^0$ is a vector of 13 elements which represents exogenous demand.[3] Combining equation 1 and 2 we get

$$\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1} \cdot (\mathbf{F}^0 + \mathbf{K}) \cdot \mathbf{y} \tag{3}$$

The implementation of the model requires an accounting table where the wine branch is explicitly reconstructed.

3 The wine flows in National Accounts

The determination of the various wine categories within national accounts is tied to the new denominations NACE REV.1 (SEC95 1995,[7]). The aggregation is based on the following criteria: the origin of grapes utilized for production, the type of wine produced and its uses. The wine output is distributed among the following branches:
01.13.1 <u>Vine growing and wine-vine firms</u>
-vine growing for wine grapes and eating grapes
-wine output from own production
01.13.5 <u>Mixed wine-vine cultivations</u>
-wine output from non-own production
15.93. <u>wine output (from non-own production)</u>
this class subdivided into two categories, includes:
-15.93.1 <u>Wine Making (special wines excluded)</u>
-wine output: table wine, v.q.p.r.d.wine (quality wines produced in predetermined regions); wine production from concentrated grape must;
-15.93.2 <u>Special Wine Making</u> -this class does not include:
wine production associated with vine growing (01.13)
wine bottling and packaging, with no transformation, 51.17 and 74.82

The adoption of this classification allows for the isolation of these branches within the Input-Output that refer exclusively to wine.[4] We will need to construct a set of rows and columns for white and red wine. In order to obtain an intersectoral table with an explicit wine sector it is necessary to break down the intermediate flows used in the various wine productions. In order to calculate the branches output, according the NACE.REV.1 classification, we need to refer to specific evaluations of wine output from own vines (agricultural firms). The availability of administrative MIPA (Minister of Agriculture and

[3]In application we assume $F^0 = 0$ in this application.

[4]The branch can be constructed on the basis of the available information, and further disaggregated according various wine typologies.

Forestry Policies) data at regional level, concerning the quantities forwarded to distillation; data from the DOC wine Committee regarding DOC and DOCG outputs, data from specific studies performed by ISMEA (Institute of Services Agriculture and Food market) has allowed for the distinction between wine production from own vines, wine production by cooperatives and wine production by vine wine industry. These data allow for an accurate analysis of the attribution of wine output to agriculture rather than to other branches (wine by cooperatives, wine by wine vine industry) through grapes acquired by agriculture. In conclusion the analysis has permitted the quantification of the new value of wine output in agriculture. This sector makes wine on his own from the 40-45 per cent of its grape production, helped by the relevant presence of cooperatives with another 40-45 per cent. From 10 to 15 per cent is produced by wine industry. These new branches have been built on unit absorptions and value added taken from survey information of the producers.

Our work concentrates on wine production within economic accounts, in order to get a greater detail degree of detail in branches 1.13.1, 15.93 and 15.94 and two different branches for wine (White and Red/Rosé). The break down takes place through coefficients applied to branches 1 and 15. The total wine output is subdivided according shares already mentioned (Ciaccia,D. 1999[3]). Since total output is known and are also known the shares of the two types of wine we can easily determine the intermediate flows. The determination of the intermediate absorptions and final demands for the Marche (Socci,C. 2003[12])takes place through the use of regional agricultural statistics compared with production technical data and households consumption data. Moreover in order to determine the destination of wine output we utilized the market shares of the branch Alcoholic beverages in the intersectoral flow table for Italy 1996.(Rampa,G. 1997[10])

The two branches under examination show relevant absorptions from agriculture (grapes for wine production), from energy water and transport sectors. For what regards the market shares output is oriented for a great part to final demand (consumption and export) and to intermediate sector transport.[5]

The greater share of value added generated by the two branches is given by other incomes. It comprises mixed income and Gross Operating Surplus.

Taxes on output show a consolidated flow which is positive but it is comprehensive of subsidies.[6]

[5]In our table this branch includes trade hotels and restaurants.
[6]The accounting table is presented in the appendix (13x13).

4 The decomposition of the structural method: the most effective demand change

We now have the data base to determine all the parameters in equation 3 that shows the interactions among industries and sectors. We can consider the direct and indirect effects of disposable incomes on industry output. Our structural matrix R will be given by:

$$\mathbf{R} = (\mathbf{I} - \mathbf{A})^{-1} \cdot (\mathbf{F}^0 + \mathbf{K}) \tag{4}$$

Its numerical determination is given in Table 1.Each cell shows the growth of the i^{th} output, $\mathbf{x}_i$, caused by a unit change income impulse, $\mathbf{y}_j$, in the j^{th} sectoral disposable income.

Table 1: Direct and indirect effects of sector disposable incomes on industry output

	I	II	III	IV	V	VI	VII
1	**-0.018**	**-0.013**	**-0.010**	**-0.007**	**-0.002**	**0.005**	**-0.007**
2	**0.005**	**0.003**	**0.003**	**0.002**	**0.001**	**-0.000**	**-0.001**
3	**0.003**	**0.002**	**0.002**	**0.001**	**0.001**	**-0.000**	**-0.000**
4	**0.005**	**0.002**	**-0.000**	**-0.001**	**-0.002**	**-0.005**	**-0.021**
5	**0.030**	**0.022**	**0.015**	**0.012**	**0.007**	**-0.004**	**-0.009**
6	**0.040**	**-0.008**	**-0.041**	**-0.079**	**-0.129**	**-0.208**	**-0.031**
7	**-0.257**	**-0.053**	**0.092**	**0.242**	**0.442**	**0.771**	**-0.076**
8	**0.047**	**0.033**	**0.022**	**0.019**	**0.014**	**-0.002**	**-0.018**
9	**0.009**	**0.006**	**0.005**	**0.004**	**0.003**	**-0.000**	**-0.002**
10	**0.088**	**0.088**	**0.100**	**0.046**	**-0.017**	**-0.065**	**-0.072**
11	**0.485**	**0.329**	**0.206**	**0.137**	**0.035**	**-0.178**	**-0.113**
12	**0.153**	**0.104**	**0.066**	**0.040**	**0.003**	**-0.068**	**0.000**
13	**0.020**	**0.015**	**0.012**	**0.009**	**0.006**	**0.000**	**1.000**

Table 1 can be easily decomposed in a sum of 7 different tables through the singular value decomposition. The decomposition is such that each sub table is "ruled" by a single scalar, called singular value, which shows the aggregated effect on the output vector of a demand vector of predetermined sectoral structure (see appendix section 1). For this reason we will refer to these singular values as macro multipliers. Matrix $\mathbf{R}$ in fact can always be written as

$$\mathbf{R} = \mathbf{U} \cdot \mathbf{S} \cdot \mathbf{V}^T \tag{5}$$

where $\mathbf{U}$ and $\mathbf{V}^T$ are two unitary matrices of convenient dimensions and $\mathbf{S}$ is a (7x7) diagonal matrix whose diagonal elements consist of the 7 scalars $\mathbf{s}_i$. Scalars $\mathbf{s}_i$ are all positive and can be ordered in decreasing order. If we denote with $\mathbf{u}_i$ the columns of matrix $\mathbf{U}$ and with $\mathbf{v}_i$ the rows of matrix $\mathbf{V}$ we can express matrix $\mathbf{R}$ as:

$$\mathbf{R} = \sum_i s_i \mathbf{u}_i \mathbf{v}_i \tag{6}$$

each of the 7 elements of the summation represents a table composing Table 1.

If the income impulse is chosen so that its structure is made equal to, say, vector $\mathbf{v}_i$ all the elements of the summation, other than $\mathbf{s}_i$, $\mathbf{u}_i$ and $\mathbf{v}_i$ become equal to zero, since vectors $\mathbf{v}_i$ (i=1,..,7) are orthogonal, and matrix $\mathbf{R}$ would reduce to:

$$\mathbf{R} = s_i \mathbf{u}_i \mathbf{v}_i \tag{7}$$

We can than say that, given our matrix $\mathbf{R}$, we are able to isolate impacts of different (aggregate) magnitude, considering that each latent macro multiplier present in matrix $\mathbf{R}$, $\mathbf{s}_i$ can be activated through a shock along the demand structure $\mathbf{v}_i$ and its impact can be observed along the output structure $\mathbf{u}_i$.

Table 2: Latent multipliers in $\mathbf{R}$.

S1	1.048	
S2	1.012	
S3	0.646	
S4	0.066	
S5	0.00	
S6	0.00	
S7	0.00	
	5.19	

Table 2 shows the macro multipliers which are present in matrix $\mathbf{R}$. Macro multiplier 1 (1.048) is the dominating one for its order of magnitude. This means that a final demand vector change produces a change on the output vector 1.048 times greater.[7] Multiplier 2 amplify the effect of the shock, while 3 reduce it. The last four macro multipliers have no effects.

Data in table 1 can be standardized taking deviations from the mean value and dividing by mean square error. In this case decomposition would produce the macro multipliers shown in table 3.

Since matrix product $\mathbf{R} \cdot \mathbf{R}^T$ represent the output correlation matrix and and that square roots of its eigenvalues are the singular values of matrix $\mathbf{R}$, we can conclude that each singular value in table 3 can be interpreted as the share of the variance related to the associated singular value. If we determine the cumulated percentage shares, we see that the first two singular values cover the 95 per cent of total variance. This means that we can confine our analysis of intersectoral

[7]Given the problems connected with aggregation in multisectoral models, this feature of singular values s_i is not of minor relevance. They are aggregated multipliers consistently extracted from a multisectoral framework and their meaning holds both if we speak in aggregated or disaggregated terms.

Table 3: Latent multipliers in **R**.

	multiplier	cumulative percentage sum
S1	3.20	0.61
S2	1.61	0.95
S3	0.37	1
S4	0.00	1
S5	0.00	1
S6	0.00	1
S7	0.00	1
	5.19	

and interindustry interaction to the first two macro multipliers to get results valid for the 95 per cent of the cases. Rather then considering matrix $\mathbf{R}$, which can be decomposed into the sum of seven impact components each one dominated a scalar multiplier

$$\mathbf{R} = \sum_{i=1}^{7} s_i \cdot \mathbf{u}_i \cdot \mathbf{v}_i \tag{8}$$

we can refer to matrix

$$\mathbf{R}^0 = s_1 \cdot \mathbf{u}_1 \cdot \mathbf{v}_1 + s_2 \cdot \mathbf{u}_2 \cdot \mathbf{v}_2 \tag{9}$$

in which addenda greater then two have been neglected. In matrix $\mathbf{R}^0$ the economic interactions are all determined by the first two aggregate impact multipliers s_1 and s_2. We note that vectors

$$s_1 \cdot \mathbf{u}_1 = \begin{bmatrix} s_1 \cdot u_{1,1} \\ s_1 \cdot u_{2,1} \\ s_1 \cdot u_{3,1} \\ . \\ . \\ . \\ s_1 \cdot u_{13,1} \end{bmatrix}, s_2 \cdot \mathbf{u}_2 = \begin{bmatrix} s_2 \cdot u_{1,2} \\ s_2 \cdot u_{2,2} \\ s_2 \cdot u_{3,2} \\ . \\ . \\ . \\ s_2 \cdot u_{13,2} \end{bmatrix} \tag{10}$$

-which split the two macro multipliers into the thirteen output sectors- represent how each of the two impact components affects the output sectors. On the other and vectors

$$s_1 \cdot \mathbf{v}_1 = \begin{bmatrix} s_1 \cdot v_{1,1} & s_1 \cdot v_{1,2} & s_1 \cdot v_{1,3} & . & . & . & s_1 \cdot v_{1,7} \end{bmatrix} \tag{11}$$

$$s_2 \cdot \mathbf{v}_2 = \begin{bmatrix} s_2 \cdot v_{2,1} & s_2 \cdot v_{2,2} & s_2 \cdot v_{2,3} & . & . & . & s_2 \cdot v_{2,7} \end{bmatrix} \tag{12}$$

-which split the same two macro multipliers into the seven institutional sectors- represent how the change in sectoral disposable income

Table 4: Impact of a disposable income on the macro multiplier

	First impact component $\mathbf{v}_1 \cdot s_1$	second impact component $\mathbf{v}_2 \cdot s_2$
I	1.861	0.240
II	1.116	0.059
III	0.556	-0.057
IV	0.118	-0.237
V	-0.493	-0.470
VI	-1.603	-0.803
VII	-1.555	1.268

Table 5: Impact of the macro multiplier on industry outputs

	First impact component $\mathbf{u}_1 \cdot s_1$	second impact component $\mathbf{u}_2 \cdot s_2$
x1	-0.874	-0.485
x2	0.990	-0.134
x3	0.990	-0.134
x4	0.796	-0.604
x5	0.990	-0.133
x6	0.742	0.671
x7	-0.651	-0.759
x8	0.966	-0.236
x9	0.986	-0.162
x10	0.937	-0.073
x11	0.996	0.081
x12	0.957	0.288
x13	-0.510	0.859

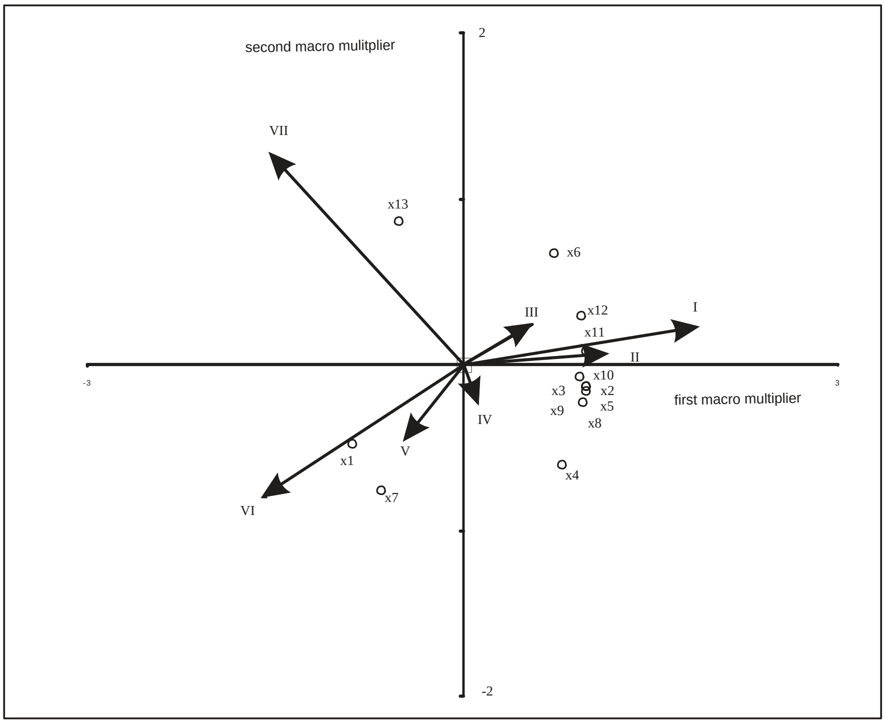
influence the two impact components. A numerical representation of these impacts for our example is given table 4 and 5

We can also give a graphical representation of each elements in the four vectors. We will define the axis of the first macro multiplier on which we measure ra the elements of vectors $s_1\mathbf{u}_1$, $s_1\mathbf{v}_1$ and the axis of the second macro multplier where we measure the elements of the vector $s_2\mathbf{u}_2$, $s_2\mathbf{v}_2$. We will then represent the couple $(s_1 v_{1,i},\ s_2 v_{1,i})$ i=1,....,7, with seven arrows showing how the change in disposable income impacts on the two macro multipliers; and couples $(s_1 u_{1,i},\ s_2 u_{1,i})$ i=1,....,13, with thirteen dots, showing how the two macro multipliers impact on sectoral output.

The modulus the arrows labelled I, II, III, IV, V, VI and VII, represent the stimulus forward to the two macro multipliers by unit change in sectoral disposable income. Dots labelled x_1 to x_{13} represent the industry effects of the macro multipliers on the industry outputs.

It has to be noted that the angle - or, better, its cosine - formed in the origin by two arrows, or by two segments connecting two dots with origin, or by arrow and a segment gives a the measure of the correlation coefficient between such two variables.

Figure 1: The interaction between final demand and output by industry.



Concentrating on the arrows we can say that there is high positive correlation among the first three income sectors. They tend to act on interaction in same direction. Income sector V tends to react in the opposed direction of the sectors I and II; as well as sector IV in respect of sector VII.

In Figure 1 we note that a set of industries, notably 2, 3, 5, 8, 9, 10 and 11 are radar concentrated with a correlation coefficient higher then 98 per cent. Which means that they receive same type of stimulus (same combination of macro multipliers). All of them and in particular wine industries, 2 and 3, perceive highly the change in disposable income of sectors I, II and III. Wines industries get more immediately the changes in disposable incomes of the lower income households. We can conclude that wine sectors are driven by the same combination of macro multipliers that rules the manufacturing sectors when stimulated by disposable income. While, agriculture perceives a higher stimulus from the same sectors but opposed direction.

5 Conclusion

The attempt of applying the social accounting framework at the regional level provides the possibility of introducing wine in a consistent system of regional accounts. The data base allows for the evaluation of policies taken the different level: local, national and EU. Some difficulties may be encountered since the accounting system related to wine is not so elaborated and much more episodic than the national one. The available evidence has been gathered and reconciled in the perspective of getting some experience on the local database and its possible integration with national data.

Impact analysis, which is the most suitable method for keeping a direct contact between data and model, can provide a tool for testing the effects of the various policies in a multi industry multi sector framework.

The application illustrated consists in finding a type of aggregation, which maintains both the multi sector and the multi industry specification of the macroeconomic variables. The method allows for the treatment of the industry and sector data, in order to determine an aggregated scale effect, which globally stimulates macroeconomic variables, though giving consistent account of their sectoral composition. Inconsistencies connected with the "historic" problem of aggregation are then overcome and a synthetic picture of interactions can be derived.

The quantification of wine interindustry interactions shows the effects of global policies in terms of the growth of the sector. Wine sector exhibits a greater similarity with other manufacturing sectors in terms of reactions to income policies while a noticeable dissimilarity is detected with agriculture and and service sectors. This phenomenon seems to confirm those empirical analyzes that allocate wine sector within the logic of manufacturing industries less and less tied to the agricultural economic behavior.

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A Spectral Decomposition

Let us consider matrix **W**, the square our (13x7) matrix **R**:

$$\mathbf{W} = \mathbf{R}^T \cdot \mathbf{R}$$

Matrix **W** has a positive definite or semidefinite square root. Given that $\mathbf{W} \geq 0$ by construction, its eigenvalues (λ_i) i=1....n shall be all real non negative (Lancaster,P. and Tiesmenetsky,M. 1985 [9]). We can define the real matrix

$$\mathbf{S} = \begin{pmatrix} \sqrt{\lambda_1} & 0 & 0 & . & 0 \\ 0 & \sqrt{\lambda_2} & 0 & . & 0 \\ 0 & 0 & \sqrt{\lambda_3} & . & 0 \\ . & . & . & . & . \\ 0 & 0 & 0 & . & \sqrt{\lambda_7} \end{pmatrix}$$

Matrix **W** is normal and hence there is a unitary matrix **V** such that

$$\mathbf{W} = \mathbf{V} \wedge \mathbf{V}^T$$

where $\wedge = S^2$. The square root of **W** shall be given by

$$\mathbf{W*} = \mathbf{VSV}^T \tag{13}$$

so that $\mathbf{W*}^2 = \mathbf{W} = \mathbf{R}^T \cdot \mathbf{R}$. The eigenvalues of matrix $\mathbf{W*} = (\mathbf{R}^T \cdot \mathbf{R})^{1/2}$ are referred as to as singular values s_1, s_2,..., s_7 of matrix **R**. Moreover the square root of first 7 eigenvalues of $\mathbf{R} \cdot \mathbf{R}^T$ coincide with the 7 singular values of **R**.

Matrix **R** can be, then, decomposed according 13. We define a matrix **S** whose elements are the singular values of $\mathbf{R}^T$ or **R**. Thus

$$\wedge = \mathbf{S}^T \cdot \mathbf{S} = \mathbf{S}^2$$

and

$$\mathbf{S}^{-1} \wedge \mathbf{S}^{-1} = \mathbf{I}$$

Define the *13*x*13* matrix:

$$\mathbf{U} = \mathbf{R} \cdot \mathbf{V} \cdot \mathbf{S}^{-1}$$

matrix **U** is orthogonal as:

$$\mathbf{U}^T \cdot \mathbf{U} = \mathbf{S}^{-1} \cdot \mathbf{V}^T \cdot \mathbf{R}^T \cdot \mathbf{R} \cdot \mathbf{V} \cdot \mathbf{S}^{-1} = \mathbf{S}^{-1} \wedge \mathbf{S}^{-1} = \mathbf{I} \tag{14}$$

from 14

$$\mathbf{U} \cdot \mathbf{SV}^T = \mathbf{R} \cdot \mathbf{V} \cdot \mathbf{S}^{-1} \cdot \mathbf{S} \cdot \mathbf{V}^T = \mathbf{R} \cdot \mathbf{V} \cdot \mathbf{V}^T = \mathbf{R}$$

Equation 1 can, then, be decomposed as

$$\mathbf{x} = \mathbf{U} \cdot \mathbf{S} \cdot \mathbf{V}^T \cdot \mathbf{y} \tag{15}$$

$\mathbf{V}$ is an (*7*x*7*) unitary matrix whose columns define the *7* reference structures for disposable income:

$$\begin{array}{l} \mathbf{v}_1 = \left[\begin{array}{cccccc} v_{1,1} & v_{1,2} & v_{1,3} & . & . & . & v_{1,7} \end{array} \right] \\ \mathbf{v}_2 = \left[\begin{array}{cccccc} v_{2,1} & v_{2,2} & v_{2,3} & . & . & . & v_{2,7} \end{array} \right] \\ \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \\ \mathbf{v}_7 = \left[\begin{array}{cccccc} v_{7,1} & v_{7,2} & v_{7,3} & . & . & . & v_{7,7} \end{array} \right] \end{array}$$

$\mathbf{U}$ is an (13x13) unitary matrix whose columns define 13 reference structures for output:

$$\mathbf{u}_1 = \left[\begin{array}{c} u_{1,1} \\ u_{2,1} \\ u_{3,1} \\ . \\ u_{13,1} \end{array} \right], \mathbf{u}_2 = \left[\begin{array}{c} u_{1,2} \\ u_{2,2} \\ u_{3,2} \\ . \\ u_{13,2} \end{array} \right], \left[\begin{array}{c} . \\ . \\ . \\ . \\ . \end{array} \right], \mathbf{u}_13 = \left[\begin{array}{c} u_{1,13} \\ u_{2,13} \\ u_{3,13} \\ . \\ u_{13,13} \end{array} \right]$$

and $\mathbf{S}$ is an (7x7) diagonal matrix of the type:

$$\mathbf{S} = \left(\begin{array}{ccccc} \mathbf{s}_1 & 0 & 0 & . & 0 \\ 0 & \mathbf{s}_2 & 0 & . & 0 \\ 0 & 0 & \mathbf{s}_3 & . & 0 \\ . & . & . & . & . \\ 0 & 0 & 0 & . & \mathbf{s}_7 \end{array} \right)$$

Scalars s_i are all real and positive and can be ordered as $s_1 > s_2 > ... > s_7$. Now we have all the elements to show how this decomposition correctly represents the macro multipliers that quantify the aggregate scale effects and the associated structures of the impact of a shock in disposable income on total output.

In fact if we express the actual vector $\mathbf{y}$ in terms of the structures identified by matrix $\mathbf{W}$, we obtain income demand vector, $\mathbf{y}^0$, expressed in terms of the structures suggested by the $\mathbf{R}$:

$$\mathbf{y}^0 = \mathbf{V} \cdot \mathbf{y} \tag{16}$$

On the other hand we can also express total output according the output structures implied by matrix $\mathbf{R}$:

$$\mathbf{x}^0 = \mathbf{U}^T \cdot \mathbf{x} \tag{17}$$

Equation 15 then becomes through equations 16 and 17:

$$\mathbf{x}^0 = \mathbf{S} \cdot \mathbf{y}^0 \tag{18}$$

which implies:

$$x_i^0 = s_i \cdot y_i^0 \tag{19}$$

where i=1,...,7. We note that matrix $\mathbf{R}$ hides seven fundamental combination of the outputs. Each of them is obtain multiplying the corresponding combination of incomes by a predetermined scalar which has in fact the role of aggregated macro multiplier.

The complex effect on the output vector of income shocks can be reduced to a multiplication by a constant s_j.

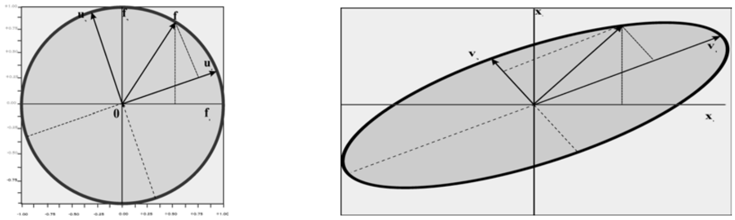
The structures we have identified play a fundamental role in determining the potential behavior of the economic system, i.e. the behavior of the system under all possible shocks. We can in fact evaluate which will be the effect on output of all income possible structures. This is easily done imposing in equation 15 a vector whose modulus is constant, say equal to one, but whose structure can assume all possible configurations. If vector $\mathbf{y}$ in equation 15 is such that

$$\sqrt{\sum_j y_j^2} = 1 \tag{20}$$

then geometrically we mean that the income vector describes a sphere of unit radius: the unit ball.It rotates around the origin, as in figure 2(a), assuming all the possible structures, including those implied by the columns of matrix $\mathbf{V}$. Correspondingly the vector of total output will describe an ellipsoid with semi-axes of length $s_1, ..., s_7$, oriented according the directions designated by the columns of matrix $\mathbf{U}$, as in figure 2(b). This ellipsoid is sometimes called the isocost of income control.

When income vector crosses a structure in $\mathbf{V}$, the vector of total output crosses the corresponding structure in $\mathbf{U}$ and the ratio between the moduli of the two vectors is given by the corresponding scalar s.

Figure 2: Unit ball and corresponding elipsoid for disposable inocme.



(a) Unit ball for disposable income (b)Corresponding elipsoid

B Data set for the application

In this section we show the Social Accounting Matrix for the Marche (1996), where the Input-Output table records the branches for the white wine and red/rosè wine.

In table the institutional sectors are:

Table 6: Institutional sectors classification

H.I.C. I	Households Income Class 0-20 (m.it.lire)
H.I.C. II	Households Income Class 20-30
H.I.C. III	Households Income Class 30-60
H.I.C. IV	Households Income Class 60-135
H.I.C. V	Households Income Class over 135
Government	Central and Local
Corporations	

The input-output branches are:

Table 7: Input-Output classification

1	Agriculture
2	White wine
3	Red and Rosè wine
4	Oil
5	Energy
6	Metal and Chemical
7	Machine and Car
8	Food
9	Tobacco and Hooch
10	Manufacury
11	Transport and Trade
12	Service market
13	Service no-market

Table 8: Social Accounting Matrix for Marche 1996

	Agricolture	white wine	red and rosy wine	oil	energy	metallic and chemical	cars and auto	alimentary products	tobacco and alcoholic	manufactured	transport and commerce	services d.v.	public services amminis	Wage and Salaries	Operating Surplus	H.I.C_I	H.I.C_II	H.I.C_III	H.I.C_IV	H.I.C_V	Corporations	General Government	Capital Account	Rest of Italy	Rest of World	
Agricolture	584,47	74,29	50,54	0,00	0,00	11,26	2,60	1212,83	72,12	221,74	13,97	4,30	61,26	0,00	0,00	42,55	91,10	259,47	161,29	13,05	0,00	0,00	-2605,16	4288,98	106,19	4667
white wine	0,17	0,02	0,01	0,00	0,00	0,18	0,00	1,55	0,62	0,21	207,08	0,00	6,75	0,00	0,00	15,45	33,08	94,21	58,56	4,74	0,00	0,00	-6,54	336,97	0,61	754
red and rosy wine	0,11	0,02	0,01	0,00	0,00	0,12	0,00	1,04	0,41	0,14	138,09	0,00	4,50	0,00	0,00	10,30	22,06	62,82	39,05	3,16	0,00	0,00	-4,36	224,71	0,41	503
oil	56,03	8,00	3,56	955,12	321,06	272,20	43,20	60,00	9,24	260,80	402,29	92,40	197,52	0,00	0,00	74,85	160,24	456,41	283,71	22,95	0,00	0,00	1171,40	594,70	88,20	5534
energy	7,43	16,92	12,13	11,04	98,60	122,25	33,60	34,95	4,92	158,65	170,85	36,50	86,47	0,00	0,00	73,59	157,54	448,71	278,92	22,57	0,00	0,00	-2746,92	3587,76	0,92	2617
metallic and chemical	208,38	29,76	13,24	16,64	45,80	3131,20	2000,51	120,50	79,85	4066,35	226,80	174,06	289,02	0,00	0,00	75,38	220,81	944,11	469,08	22,63	0,00	0,00	333,95	3308,29	883,50	16660
cars and auto	37,11	4,30	2,36	1,30	39,40	115,40	1596,26	1438,25	0,58	946,00	253,00	7,50	703,72	0,00	0,00	31,45	246,54	1652,31	672,04	8,20	0,00	0,00	6249,88	-1538,65	2910,54	15377
alimentary products	421,14	60,15	26,75	0,10	0,00	737,65	0,00	810,85	4,45	893,95	3379,01	11,77	168,10	0,00	0,00	262,12	561,12	1598,23	993,46	80,38	0,00	0,00	11,65	2138,38	90,45	12250
tobacco and alcoholic	0,40	0,06	0,03	0,00	0,00	0,43	0,00	3,68	1,46	0,50	613,51	0,00	15,97	0,00	0,00	36,55	78,24	222,84	138,52	11,21	0,00	0,00	-15,46	673,38	1,44	1783
manufactured	34,36	4,91	2,18	11,84	70,43	600,00	590,46	142,40	72,70	7504,02	1770,50	130,00	665,22	0,00	0,00	299,93	878,64	3756,76	1866,55	90,06	0,00	0,00	8275,27	5143,82	4493,88	36404
transport and commerce	396,88	56,69	25,21	47,54	82,51	1656,40	1607,96	649,40	139,71	3501,40	7017,99	681,16	1041,48	0,00	0,00	1208,79	2587,67	7370,38	4581,43	370,67	0,00	0,00	-151,02	6908,23	1070,20	40851
services d.v.	0,09	0,01	0,01	0,71	12,56	28,80	628,91	0,70	1,69	7,80	13,60	0,00	0,00	0,00	0,00	254,97	545,81	1554,62	966,35	78,18	0,00	0,00	-575,99	0,00	1,50	3520
public services amminist.	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	37,75	80,81	230,18	143,08	11,58	0,00	8530,20	193,60	0,00	3,60	9231
Wages and Salaries	239,87	70,07	43,40	128,82	680,41	1223,99	1211,01	708,76	10,29	3726,09	5444,09	578,67	5698,21	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	175,87	19940
Operating surplus	1158,49	227,90	152,31	205,20	169,79	995,66	714,85	344,99	17,39	5835,36	15397,56	1387,50	292,56	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	26900
___H.I.C_I	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	133,31	529,26	41,86	0,00	0,00	0,00	0,00	414,52	1271,93	0,00	13,38	2,41	2407
___H.I.C_II	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	2005,84	875,43	0,00	4,14	0,00	0,00	0,00	1375,43	2533,58	0,00	1,33	1,04	6797
___H.I.C_III	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	10378,92	5302,79	0,00	0,00	16,61	0,00	0,00	3988,45	6652,93	0,00	5,31	6,73	26352
___H.I.C_IV	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	6998,34	6244,45	0,00	0,00	0,00	3,07	0,00	3308,53	3400,46	0,00	0,98	3,78	19960
___H.I.C_IV	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	363,90	1450,94	0,00	0,00	0,00	0,00	0,00	250,65	606,38	0,00	0,00	0,45	2672
Corporations	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	12128,37	40,51	159,89	553,34	380,47	12,39	3637,36	5720,12	0,00	0,00	1791,29	24424
General Government	-212,68	171,26	118,01	1704,24	74,20	92,60	268,86	108,70	557,65	2159,10	1568,80	156,96	0,00	0,00	368,32	203,18	1084,95	5344,27	4042,42	408,22	2585,33	0,00	0,00	3754,83	4158,31	28718
Taxes on Import	44,26	0,71	1,28	74,55	1,16	26,44	149,12	32,00	18,74	110,83	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	459
Taxes on products Net	-257,88	170,55	116,73	1629,69	73,00	66,16	119,74	76,71	537,87	2048,27	1568,80	156,96	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	6307
Taxes on income	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	160,57	474,63	2205,01	1925,16	298,63	2241,77	0,00	0,00	0,00	0,00	7306
Social Contributions	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	40,15	604,06	3125,61	2107,55	109,59	0,00	0,00	0,00	0,00	0,00	5987
Capitla formation	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-541,76	-358,53	1472,33	4525,84	1059,51	2725,57	0,00	0,00	3109,12	-1861,74	10130
Rest of Italy	1177,40	23,60	42,56	1243,76	977,30	5209,84	4009,74	5658,23	736,25	4867,56	2678,68	41,10	0,00	0,00	0,00	239,20	242,69	314,14	355,77	452,83	4280,87	0,00	0,00	0,00	0,00	32552
Rest of World	558,14	5,72	10,31	1207,60	45,31	2461,86	2669,52	952,86	74,45	2254,26	1554,86	218,42	0,00	59,24	0,00	0,00	0,00	0,00	0,00	0,00	1857,02	0,00	0,00	0,00	0,00	13930
	4668	754	503	5534	2617	16660	15377	12250	1784	36404	40851	3520	9231	19940	26900	2407	6797	26352	19960	2672	24424	28716	10130	32552	13930	