Macro Multipliers approach to output generation and income distribution in a Social Accounting framework

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Abstract

The aim of the paper is to analyze the features and the changes in the personal income distribution in a region of Italy within a structural framework.

In this paper the attempt is made to provide a method of analysis that can give a further insight into the interactions among industries and institutional sectors. An application that relies on a regional data base, inspired by the Social Accounting Matrix, illustrates how macro-multipliers ruling the multi-sector multi-industry interactions can be defined and evaluated. This feature greatly helps in showing the impact of the structure of macroeconomic variables since all the possible behavior of the economy are determined by those multipliers: either those patterns that have emerged, because have been activated by the actual shock, and those that have kept latent.

The identification of macro multipliers allows for the consistent definition of forward and backward dispersion, a tool especially efficient in the study of propagation since it is not confined to predetermined structures of macroeconomic variables and still allows for the determination of "summary" measures of dispersion through industries and sectors.

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1 Introduction

Some decades ago national accounts suffered from a fundamental dichotomy between income-final demand accounts and output-interindustry flows accounts. As Richard Stone pointed out (Stone, 1979) we faced two different and separated accounting systems which acted independently with almost no information exchange.

In the last decades the national accounting schemes have completely realized the integration of the two aspects and one side provides the information support to the other, causing the progressive integration in the actual accounting practice. Though integrated, the accounting scheme remains flexible and open. Its matrix representation (National Accounting Matrix) constitutes a consistent nucleus that can be extended according the aims of the research. The Social Accounting Matrix is the result of this expansion that can be moved forward to include a greater set of economic and social phenomena at a substantial degree of detail.

From the SAM approach emerges a model of circular income flow which is more articulated than the usual one: each macroeconomic flow variable, conveniently disaggregated, generates a second flow variable through the use of a structural matrix and progressively so until the loop is closed. Final demands determine total outputs and value added by industry; the latter generates domestic incomes by factor which compose disposable incomes by institutional sectors; these give rise to final demands closing the loop (Miyazawa, 1970).

For facing these progresses in the design of a data base which provides meaningful sectorization of the major macroeconomic variables, flexible tools of analysis are needed, to get a deeper insight in the propagation phenomena characterizing sectoral and industrial interactions. In these phenomena the scale, but especially, the structure of macroeconomic variables play a major role. The traditional tools for studying propagation are those provided by impact multipliers and linkage analysis. These tools, however, design procedures that do not give a complete account of the effects of the changing structures of macro-variables.

The propagation analysis we propose is based on a decomposition that allows for the identification and quantitative determination of aggregated macro multipliers, which lead the economic interactions, and the structures of macroeconomic variables, that either hide or activate these forces. The analysis will be applied to an extended income-output loop that can be quantitatively tested forwarding a shock on a given macro-variable and observing the effects on another macro-variable within the loop. It will identify the most efficient struc-

ture, without confining on the equi-distributed unitary shock. "Summary" measures will be found, consistent with the multi-sectoral and multi-industry framework, that will allow to measure the degree of interaction among sector and industry components.

In section 2 the discussion on impact multipliers and linkage analysis is briefly referred to, in order to restate the "statistical" purpose of summary measures of linkage. In section 3 we describe the data base for our application, that we obtained integrating data from various sources in an accounting scheme, inspired to the SAM, where institutional sectors are articulated into various income classes. Section 4 shows the formalization of an extended circular flow loop on which the analysis will be performed. In section 5 the deterministic analysis of propagation is performed in order to identify and quantify all the macro multipliers that rule the economic interactions. Section 6 proposes a "statistical" approach where the structural matrix is conveniently refined to obtain a synthetic representation of the interactions. In the same section measures of backward and forward dispersion are stated with reference to the dominating macro multipliers. A summary representation is then provided, which relies on the concepts of backward and forward dispersions, with the aim of determining the strength of multi-sector multi-industry interactions.

Appendix A shows the singular values decomposition related to eigenvalues decomposition. Appendix B shows a brief sketch of a SAM for the Marche region, showing the criteria followed in disaggregation the institutional sector households into income classes.

2 Some considerations on multiplier and linkage analysis

The original Input-Output (I-O) problem consists in the search for an equilibrium output vector for the n I-O sectors of the economy. Since in the following section income will be disaggregated by institutional sectors, in order to avoid misinterpretation, we will use the term industries for producing sectors, and the sector for institutional sectors. Such vector conveniently faces the predetermined final demand vector $\mathbf{f}$ by industries, and the induced industrial demand. The equilibrium output vector is given by

$$\mathbf{x} = \mathbf{R} \cdot \mathbf{f} \tag{1}$$

where $\mathbf{R} = [\mathbf{I} - \mathbf{A}]^{-1}$ and $\mathbf{A}$ is the technical coefficients matrix, and generally exists, as in general the technology can be expected to be productive, i.e. the technology is such that a part of total output is still available for final uses, after the intermediate requirements have been satisfied. In this case, $\mathbf{A}$ satisfies the Hawkins-Simon conditions. The

Figure 1: Inter-industry output flows
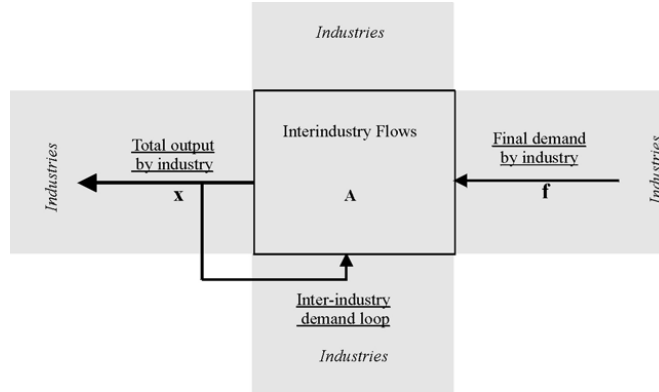
R matrix is usually referred to as the Leontief multipliers matrix and its elements, r_{ij}, show the direct and indirect requirements of industry output i per unit of final demand of product at industry j. Extensive use is made of matrix **R** within the traditional multipliers analysis and a substantial part of linkage and key sectors analysis is based on it. **R** matrix provides, in fact, a set of disaggregated multipliers that are recognized to be most precise and sensitive for studies of detailed economic impacts. These multipliers recognize the evidence that total impact on output will vary depending on which industries are affected by changes in final demand. The ith total output multiplier measures the sum of direct and indirect input requirements needed to satisfy a unit final demand for goods produced by industry i. Input-Output multipliers can be derived from either an open I-O model and a partially closed I-O model. The first set includes type I and type II multipliers. For the determination of type I multiplier all components of final demand are treated exogenously. Type I multiplier will then represent the ratio of direct and indirect output changes to the initial direct change in final demand.

Multipliers can be however determined taking one or more components of final demand as endogenous. If the only final demand component to be treated endogenously is personal consumption expenditures, the multipliers are referred to as type II. In this case the model is said to be partially closed with respect to households. Each type II multiplier will then represent the ratio of the direct indirect and induced changes to the initial direct change. If another final demand component such as state and local government expenditure is also treated endogenously the multiplier is referred as type III (Lee,

1986).

When a final demand component is made endogenous the corresponding part of value added must also be treated endogenously; consequently, personal consumption expenditures have counterpart in value added referred to as wages and salaries; state and local government expenditures a counterpart referred to as taxes etc. The inverse coefficients of the augmented matrix reflect the induced effects of changed incomes on final outputs. Finally income and employment multipliers, type IV multipliers, can be obtained by premultiplying the matrix of output multipliers by a row vector of wage to output ratios in the case of income, and employment to output ratio in case of employment (Polenske and Jordan, 1988).

Research on linkage analysis dates back to the definitions elaborated by Rasmussen of "summary measures for the inverse matrix" (Rasmussen, 1956). Developments in research have provided various definitions of linkage (Hirschman, 1958) which have led to the indicators called nowadays "forward linkages" and "backward linkages". These indicators are applied to the technical coefficients matrix, to the Leontief inverse or to the matrix of constant market shares (Ghosh, 1958) according the purposes of the research. However, from a modelling viewpoint, the fixed technical coefficient assumption is conflicting with the constant market shares hypothesis, since a model based on fixed technical coefficients will imply non constant market shares, and a model with constant market shares will imply varying technical coefficients.

This is the reason why we will confine ourselves to the *leontievian* approach based on the concept of fixed technical coefficients and refer to the origins through the Rasmussen definitions. He noted that the sum, $r_{.j}$, of ith column elements

$$r_{.j} = \sum_{i=1}^{m} r_{ij} \qquad (2)$$

corresponds to the total increase in output from the whole system of industries needed to match an increase in the final demand for the product of industry j by one unit.
Similarly the sum, $r_{i.}$, of row elements i.e.

$$r_{i.} = \sum_{j=1}^{m} r_{ij} \qquad (3)$$

gives the increase in output of industry i needed, in order to cope with a unit increase in the final demand for the product of each industry.

We can take the average, of $r_{.j}$, and they will represent an estimate of the (direct and indirect) increase in output to be supplied by an industry chosen at random if final demand for the products of industry j expands by one unit:[1]

$$\left(\frac{1}{m}\right) \cdot r_{.j} \qquad (j = 1, 2, .., m) \qquad (4)$$

Similarly

$$\left(\frac{1}{m}\right) \cdot r_{i.} \qquad (i = 1, 2, .., m) \qquad (5)$$

can be regarded as the average increase in output to be supplied by industry i if the final demand for the products of an industry chosen at random is increased by one unit.

For performing consistent interindustry comparisons, we need to normalize these averages by the overall average defined as

$$\frac{1}{m^2} \sum_{j=1}^{m} \sum_{i=1}^{m} r_{ij} = \frac{1}{m^2} \sum_{j=1}^{m} r_{.j} = \frac{1}{m^2} \sum_{i=1}^{m} r_{i.} \qquad (6)$$

and thus consider the indices

$$\pi_j = \frac{\frac{1}{m} \cdot r_{.j}}{\frac{1}{m^2} \cdot \displaystyle\sum_{j=1}^{m} r_{.j}} \qquad (7)$$

and

$$\tau_i = \frac{\frac{1}{m} \cdot r_{i.}}{\frac{1}{m^2} \cdot \displaystyle\sum_{i=1}^{m} r_{i.}} \qquad (8)$$

The aim of the direct and indirect backward linkage index π_j, the *power of dispersion* in the Rasmussen definition [2], is to measure the potential stimulus to other activities from a demand shock in any industry j. If $\pi_j > 1$ an industry will need a comparatively large production increase to meet a unit increase in final demand for the products of industry j. When $\pi_j < 1$ industry j relies heavily on the system of industries and vice versa. π_j can be considered an index of the power of dispersion for industry j. This index describes the relative extent to which an increase in final demand for the products of industry j is dispersed throughout the system of industries. The index also expresses the extent of the expansion caused in the system of industries by expansion in industry j.

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[1]Rasmussen (1956)ibidem p.130.
[2]Rasmussen (1956) ibidem p.135.

The forward linkage τ_i, the *sensitivity of dispersion* in the Rasmussen definition, measures the degree at which on industry output is used by other industries as an input. In the case of $\tau_i > 1$, for given increases in demand, industry i will have to increase its output more than other industries. Index τ_i is an index of sensitivity of dispersion for the industry i. This index expresses the extent to which the system of industries relies on industry i and the degree to which industry i is affected by an expansion in the system of industries.

It has to be stressed, however, that the Rasmussen definitions were of statistical nature, since both measures were mean values of either outputs or final demands of industries chosen at random. For each of these measures, in fact, he elaborated a coefficient of variation in fact a standard deviation. In particular, for the power of dispersion we get

$$\sigma_{.j} = \frac{\sqrt{\frac{1}{m-1}\sum_{i=1}^{m}(r_{ij} - \sum_{i=1}^{m}r_{ij})^2}}{\frac{1}{m}\sum_{i=1}^{m}r_{ij}} \qquad (j = 1, \ldots, m) \qquad (9)$$

and for the sensitivity of dispersion:

$$\sigma_{i.} = \frac{\sqrt{\frac{1}{m-1}\sum_{j=1}^{m}(r_{ij} - \sum_{j=1}^{m}r_{ij})^2}}{\frac{1}{m}\sum_{j=1}^{m}r_{ij}} \qquad (i = 1, \ldots, m) \qquad (10)$$

Nevertheless the original statistical approach of the Rasmussen analysis progressively disappeared and the interpretation of his measures have definitely become deterministic.

It has to be stressed,however, that all these measures, built starting from matrix $\mathbf{R}$, are not independent of the structure of the either total output vector, neither which we observe the effects, nor of the structure of final demand vector on which we impose the unit demand shock.

The column sum of the $\mathbf{R}$ matrix in equation [1] implies the consideration of a set of final demand vectors of the type:

$$f^1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ . \\ 0 \end{bmatrix}, f^2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ . \\ 0 \end{bmatrix}, \cdots, f^m = \begin{bmatrix} 0 \\ 0 \\ 0 \\ . \\ 1 \end{bmatrix} \qquad (11)$$

while the sum of row elements in equation [1] implies the consideration of a final demand structure of the type:

$$f = \begin{bmatrix} 1 \\ 1 \\ 1 \\ . \\ 1 \end{bmatrix} \qquad (12)$$

We can expect that these measures hold for demand vectors of varying scale but with the same structures of equations [11] or [12]. However neither the demand vector nor its changes will ever assume a structure of this type. This is why some authors come to the drastic conclusion that "linkage should be never used" (Skolka, 1986).

On the other hand it is a common opinion that the structure of final demand produces the most different effects on the level of total output (Ciaschini, 1988c). Given a set of non zero final demand vectors, whose elements sum up to a predetermined level, but with varying structures, we will have to expect that the corresponding level of total output will also vary considerably.

For these reasons we cannot confine our knowledge of the system to the picture emerging from measures which can only show what would happen if final demand assumed a predetermined and unlikely structure.

3 The data base: Towards a Social Accounting Matrix for Marche

The basic organization of the data base that has been built, is inspired by social the accounting matrix scheme and follows the matrix presentation of regional economic accounts. The income circular flow is quantified and connects data on the production process (final demand, total output and value added generation) gathered by branches which play the role of industries, with data on the distribution process (factor allocation of value added, primary and secondary distribution of incomes) collected by institutional sectors.

The main attempt is that of integrating data on income distribution and final demand by income classes considered as sub-sectors of households. The scarcity of official statistics at the local level implies the reference to heterogeneous data sources and the attempt to solve problems connected with the definition of aggregates and the data gathering methodology.

For this aim a hierarchy of the data sources has been determined for constructing the accounting table. This hierarchy implies the

definition of a reliability scale of the organizations in charge of data collection, such as the Central Statistical Office and Central and local Administration, which will reveal useful in the phase of data balancing. Balancing, in fact has been obtained through the method of constrained generalized Least Squares (Stone, 1979), that requires the exogenous definition of a variance covariance matrix of observed data.

The accounting arrangement of the data flows goes through the construction of a National Accounting Matrix, bi-regional NAM, which is a presentation of the *T accounts* in table form, based on the bi-regional Input-Output Table (TEI) of Marche by IRPET for 1996 and the regional economic accounts, provided by ISTAT 1996, (ISTAT, 1996). On this basis the disaggregation into institutional sectors (and sub-sectors) of income distribution has been performed. In this way a first version of a simplified bi-regional Social Accounting Matrix for Marche (M) and the rest of Italy (rI) [3] has been obtained which constitutes the basis for further developments and extensions. These extensions will concern the further integration of social information which is not included in National Accounts.

The Matrix can be broken up into quadrants which can be further divided into blocks. A brief sketch of blocks in each of the six sub matrices, as shown in Table [1], can be easily described as follows:

- quadrant I - production, primary allocation, secondary distribution and capital formation blocks in region M;

- quadrant II - production, secondary distribution of incomes entering in region M;

- quadrant III - production, secondary distribution of incomes entering in region rI;

- quadrant IV - production, primary allocation, secondary distribution and capital formation blocks in region rI;

- quadrant V - production, primary allocation, secondary distribution and capital formation blocks referred to Public Administrations;

- quadrant VI - operations with the rest of the world block.

Accounts are given in rows and columns corresponding to eight denominations namely Output, Wage and Salaries, Other Incomes, Households, Corporations, Capital formation, Public Administrations and Rest of the World.

The accounting reconciliation of the flows in the NAM implies various problems connected to the lack of detail in the economic accoun-

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[3]See SNA'94 chapter XX, p.464 (United Nation, 1994).

Table 1: Biregional *NAM* table for the whole economy

		Area *M*						Area *rI*						Public Administration	Rest of the World	Totals
		Output	Wages & Salaries	Other incomes	Households	Corporations	Capital Formation	Output	Wages & Salaries	Other incomes	Households	Corporations	Capital Formation			
Area *M*	Output	$B^{M,M}$			$C^{M,M}$		$I^{M,M}$	$B^{M,rI}$			$C^{M,rI}$		$I^{M,rI}$	$C^{M,PA}$	E^M	Q_1
	Wages & Salaries	$Va_1^{M,M}$													$T^{M,RdM}$	Q_2
	Other incomes	$Va_2^{M,M}$														Q_3
	Househols		$Ld^{M,M}$	AR	$T^{M,M}$	$T^{M,M}$					$T^{M,rI}$	$T^{M,rI}$		$T^{M,PA}$	$T^{M,RdM}$	Q_4
	Corporations			AR	$T^{M,M}$	$T^{M,M}$					$T^{M,rI}$	$T^{M,rI}$		$T^{M,PA}$	$T^{M,RdM}$	Q_5
	Capital formation				$S^{M,M}$	$S^{M,M}$							$A^{M,rI}$	$S^{M,PA}$	$A^{M,RdM}$	Q_6
Area *rI*	Output	$B^{rI,M}$			$C^{rI,M}$		$I^{rI,M}$	$B^{rI,rI}$			$C^{rI,rI}$		$I^{rI,rI}$	$C^{rI,PA}$	E^{rI}	Q_7
	Wages & Salaries							$Va_1^{rI,rI}$							$T^{rI,RdM}$	Q_8
	Other incomes							$Va_2^{rI,rI}$								Q_9
	Households				$T^{rI,M}$	$T^{rI,M}$			$Ld^{rI,rI}$	$AR^{rI,rI}$	$T^{rI,rI}$	$T^{rI,rI}$		$T^{rI,PA}$	$T^{rI,RdM}$	Q_{10}
	Corporations				$T^{rI,M}$	$T^{rI,M}$				$AR^{rI,rI}_m$	$T^{rI,rI}$	$T^{rI,rI}$		$T^{rI,PA}$	$T^{rI,RdM}$	Q_{11}
	Capital formation						$A^{rI,M}$				$S^{rI,rI}$	$S^{rI,rI}$		$S^{rI,PA}$	$A^{rI,RdM}$	Q_{12}
Public Adminstration		$T^{PA,M}$		$AR^{PA,M}$	$T^{PA,M}$	$T^{PA,M}$		$T^{PA,rI}$		$AR^{PA,rI}$	$T^{PA,rI}$	$T^{PA,rI}$		$T^{PA,PA}$	$T^{PA,RdM}$	Q_{13}
Rest of the World		Imp	$T^{RdM,M}$	$T^{RdM,M}$	$T^{RdM,M}$	$T^{RdM,M}$		Imp		$T^{Rdm,rI}$	$T^{Rdm,rI}$	$T^{Rdm,rI}$		T^{RdMPA}		Q_{14}
Totals		Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8	Q_9	Q_{10}	Q_{11}	Q_{12}	Q_{13}	Q_{14}	

ting of institutional sectors. We need to add to this the lack of information in regional economic data, which will be hopefully attenuated when the SEC 95 (EUROSTAT, 1996) revision will begin to give results. In this application we confine to data in quadrant I and then we will consider the second region on an aggregated basis, pointing the attention to the inner flows in Marche. Quadrant I in Table [1], then, gives account of the internal flows of the region and their allocation in different blocks in order to describe the whole circular flow. In the application that follows we have concentrated on the flows relative to Marche, we then refer to a simplified version of Sam, extracted from the SAM, shown in Table [2] that is shows as follows: Table [2] gathers data from 11 input output sectors [Agriculture, Oil, Energy, Metal & Chemical Products, Machinery and Cars, Food, Tobacco & Alcoholic Beverages, Manufacturing, Trade Transportation, Marketable Service, Non Marketable Services], 7 institutional sectors[4] [I_Income class, II_Income class, III_Income, class IV_Income class, V_Income class, Corporations, Public Administration], 3 value added components [Wage and Salaries, Other incomes, Indirect Tax], 2 macro sectors [Rest of Italy, Rest of the world]. The numerical determination of the table in

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[4]The Households Income Class are disaggregated for disposable income.

Table 2: Simplified version of Sam

		Industries 11	Va components 3	Sectors 7	Rest of Italy 1	Rest of World 1
Industries	11	$\mathbf{A}$	-	$\mathbf{F}^{IO,SI}$	$\mathbf{E}^{rI}$	$\mathbf{E}^{RoW}$
Va components	3	$\mathbf{V}^{IO}$	-	-	-	-
Sectors	7	-	$\mathbf{V}^{SI}$	$\mathbf{T}^{SI}$	$\mathbf{T}^{rI}$	$\mathbf{T}^{RoW}$
Rest of Italy	1	$\mathbf{M}^{rI}$	-	$\mathbf{T}$	-	-
Rest of World	1	$\mathbf{M}^{RoW}$	-	$\mathbf{T}$	-	-

shown in appendix B.

4 The extended output income circular flow

The results attained in social accounting encourage the attempt of building an extended version of the income circular flow where the interactions between industries and institutions could be specified and evaluated.

Figure 2: Interindustry and intersectoral flows
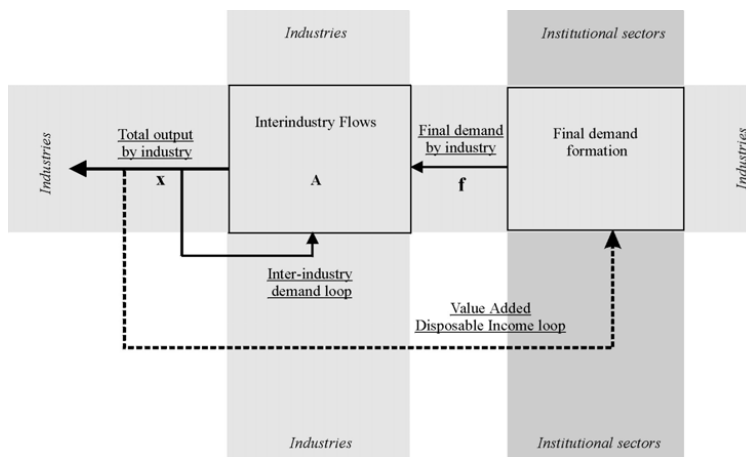
As shown in figure [2], with the dotted arrow, the income distribution process creates a feedback loop between industry outputs and final demand. This loop is built through various logical phases. The production process, that takes place at industry level, generates total output, $\mathbf{x}$, and gross value added by the 11 I-O industries, $\mathbf{v}(\mathbf{x})$,

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(Gross value added generation). Value added by I-O industry is than allocated to the 3 value added components (factors), $\mathbf{v}^c(x)$ (Gross value added allocation). Value added by components is then allocated to the 7 institutional sub-sectors, $\mathbf{v}^c(x)$ (Primary distribution of income). Value added by institutional sectors is then redistributed among them through taxation to generate disposable incomes by the 7 institutional sub-sectors, $\mathbf{y}(\mathbf{x})$ (Secondary distribution of income). Finally disposable income will generate final demand by institutional sub-sectors which will be transformed into final demand by IO industries, $\mathbf{f}(\mathbf{x})$ (Final demand formation).

On this logical sketch of the extended circular flow, we can define the structural parameters representing the distribution matrices. Of course we know that a simple model is hard to be accepted in these times of computable general equilibrium. However we think that a simple one, sometimes, might let an easier data interpretation and we are encouraged by Leontief recommendations: the model conceptually placed near the data base and no econometrics, if possible.

Our distributive structural matrices will be given by
Gross value added generation(by industry)

$$\mathbf{v}(x) = \mathbf{L} \cdot \mathbf{x} \tag{13}$$

where $\mathbf{L}[11,11]$ gives the value added by industry starting from the output vector and technical coefficients matrix.

Gross value added allocation(by VA components)

$$\mathbf{v}^c(x) = \mathbf{V} \cdot \mathbf{v}(x) \tag{14}$$

where $\mathbf{V}[3,11]$ represents the distribution of value added to the factors (components).

Primary distribution of income(by Institutional sub-sectors)

$$\mathbf{v}^{si}(x) = \mathbf{P} \cdot \mathbf{v}^c(x) \tag{15}$$

where $\mathbf{P}[7,3]$ represents the distribution factors' value added income to the sectors.

Secondary distribution of income(by Institutional sub-sectors)

$$\mathbf{y}(x) = (\mathbf{I} + \mathbf{T}) \cdot \mathbf{v}^{si}(x) \tag{16}$$

where $\mathbf{T}[7,7]$ represents net income transfers among sub-sectors.

Final demand formation(by industry)

$$\mathbf{f}(x) = \mathbf{F}^0 \cdot \mathbf{y}(x) + \mathbf{K} \cdot \mathbf{y}(x) + \mathbf{f}^0 \tag{17}$$

where $\mathbf{F}^0$ provide the consumption demand structure by industry and is given by the product of two matrices, $\mathbf{F}^0 = \mathbf{F}^1 \cdot \mathbf{C}$, where $\mathbf{F}^1$ [11,7]

transforms the consumption expenditure by institutional sector into consumption by industry and $\mathbf{C}[7,7]$ represents the consumption propensities by institutional sector.

$\mathbf{K}$ represents the investment demand and is given by $\mathbf{K} = \mathbf{K1} \cdot s \cdot (\mathbf{I} - \mathbf{C})$ where $\mathbf{K1}[11,7]$ represents the investment demands to I-O industry and scalar s represents the share of private savings which is transformed into investment i.e. "active savings"; $\mathbf{f}^0$ is a vector of 11 elements which represents exogenous demand.

If we put $\mathbf{F} = [\mathbf{F}^0 + \mathbf{K}]$ equation[17] becomes

$$\mathbf{f}(x) = \mathbf{F} \cdot \mathbf{y}(x) + \mathbf{f}^0 \qquad (18)$$

substituting through the equations [13][17] in 18 we get

$$\mathbf{f}(x) = \mathbf{F} \cdot [\mathbf{I} + \mathbf{T}] \cdot \mathbf{P} \cdot \mathbf{V} \cdot \mathbf{L} \cdot \mathbf{x} + \mathbf{f}^0 \qquad (19)$$

We now turn to the output generation process which is ruled by the Leontief model.

Output generation

$$\mathbf{x} + \mathbf{m} = \mathbf{A}(x) + \mathbf{f}(x) \qquad (20)$$

where $\mathbf{m}$ represents imports, $\mathbf{A}$ the technical coefficients matrix, $\mathbf{f}(x)$ represents the demand vector.

Imports can be modelled according its main components, intermediate consumptions, endogenous demand and exogenous demand:

Import

$$\mathbf{m} = \mathbf{A}^m \cdot (\mathbf{x}) + \mathbf{F}^m \cdot (\mathbf{I} + \mathbf{T}) \cdot \mathbf{P} \cdot \mathbf{V} \cdot \mathbf{L} \cdot \mathbf{x} + \mathbf{f}^m \qquad (21)$$

where $\mathbf{A}^m[11,11]$ represents the intermediate imports matrix, $\mathbf{F}^m[11,7]$ represents import shares of endogenous demands and $\mathbf{f}^m$ represents imports generated by an exogenous shock.

Substituting the equations [19] and [21] in [20] we finally get:

$$\mathbf{x} = [\mathbf{I} - (\mathbf{A} - \mathbf{A}^m) - (\mathbf{F} - \mathbf{F}^m) \cdot (\mathbf{I} + \mathbf{T}) \cdot \mathbf{P} \cdot \mathbf{V} \cdot \mathbf{L}]^{-1} \cdot (\mathbf{f}^0 - \mathbf{f}^m) \quad (22)$$

Figure 3 shows a diagram where the fundamental mechanism of production and distribution is shown in terms of interaction between industries, sectors and factors (value added components).

In Figure [3] each arrow identifies an expenditure flow while each box a matrix transformation of a flow variable into another. In the upper part the inter industry demand loop in Figure [3] can be recognized.

The extended output-income circular flow emerging in Figure [3] allows for an extension of the study of the propagation. We can choose, in fact, on which flow variable to act with a unit shock and on which

Figure 3: extended output income circular flow
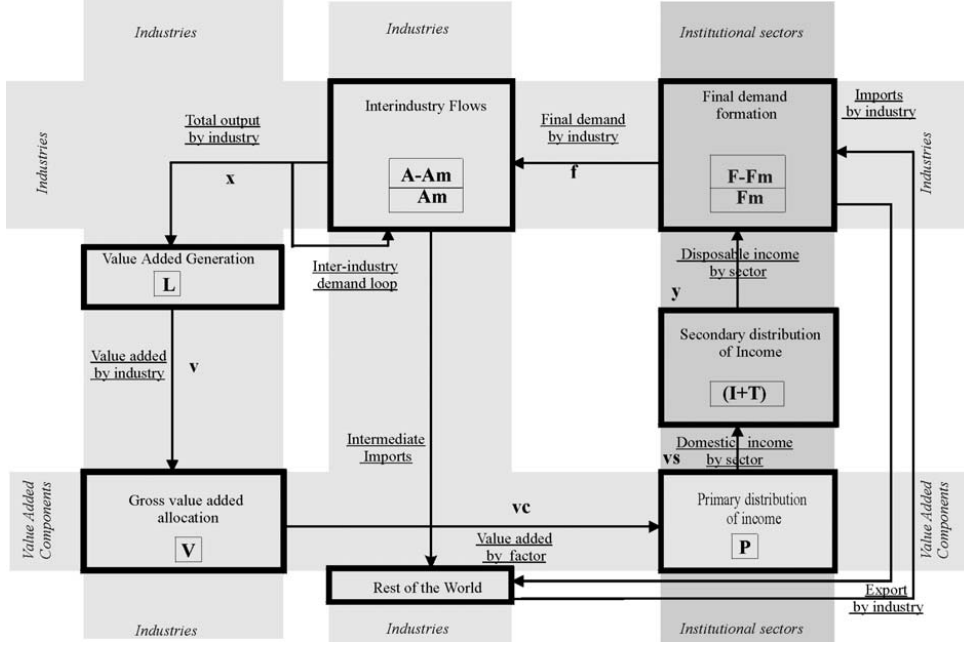
variable to observe the effects. For each flow variable we need to specify an order of magnitude, i.e. the scale and a composition, i.e. the structure. If we want to impose a unit shock on final demand and observe its propagation on domestic output we need to refer to equation [22], but other arrangements of structural matrices are easily found if we need to impose a shock on, say, income redistribution and observe it on value added by factor.

5 Searching for the most effective demand change: The decomposition of the structural relationship between final demand and output

The direct and indirect effects of final demand on total output are then quantified in our structural matrix $\mathbf{R}$

$$\mathbf{R} = [\mathbf{I} - (\mathbf{A} - \mathbf{A}^m) - (\mathbf{F} - \mathbf{F}^m) \cdot (\mathbf{I} + \mathbf{T}) \cdot \mathbf{P} \cdot \mathbf{V} \cdot \mathbf{L}]^{-1} \qquad (23)$$

Its numerical determination is shown in Table [3]. Each cell shows the growth of the ith industry output, $\mathbf{x}_i$, caused by a demand impulse of 1, $\mathbf{f}_i$, in the demand of goods produced by the ith industry. The

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twelfth column shows the row sum which represent the total effect on the ith industry output of a unitary final demand shock, $\mathbf{x}$, shown in column thirteenth, $\mathbf{f}$. The last row presents the column sums of the table and gives the effect on all the industry on outputs of a unit change in demand of goods produced by the ith industry.

Table 3: Direct and indirect effects of final demand on total output

	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}	f_{11}	Sum	f
x_1	1.28	0.09	0.06	0.04	0.04	0.21	0.20	0.07	0.08	0.08	0.06	2.22	1
x_2	0.11	1.23	0.25	0.08	0.08	0.07	0.18	0.11	0.12	0.15	0.11	2.46	1
x_3	0.04	0.05	1.04	0.02	0.03	0.02	0.06	0.04	0.04	0.05	0.04	1.43	1
x_4	0.32	0.42	0.27	1.22	0.40	0.17	0.52	0.40	0.33	0.39	0.23	4.67	1
x_5	0.51	0.65	0.34	0.26	1.38	0.24	0.72	0.46	0.54	0.57	0.27	5.93	1
x_5	0.26	0.27	0.17	0.11	0.12	1.19	0.33	0.20	0.22	0.22	0.18	3.26	1
x_7	0.03	0.05	0.04	0.02	0.02	0.02	1.06	0.03	0.04	0.04	0.03	1.39	1
x_8	0.98	1.27	0.73	0.55	0.61	0.50	1.44	2.07	1.10	1.16	0.55	10.95	1
x_9	1.42	1.98	1.36	0.95	1.05	0.88	2.30	1.44	2.63	1.81	1.29	17.10	1
x_{10}	0.16	0.25	0.18	0.10	0.12	0.10	0.27	0.16	0.19	1.20	0.16	2.88	1
x_{11}	0.15	1.18	0.58	0.25	0.31	0.22	1.19	0.43	0.39	0.45	1.49	6.64	1
Sum	5.28	7.44	5.02	3.6	4.16	3.62	8.27	5.40	5.67	6.11	4.41	58.975	11

Table [3] can be easily decomposed in a sum of 11 different tables through the singular value decomposition. The decomposition is such that each sub table is "ruled" by a single scalar which shows the aggregated effect on the output vector of a demand vector of predetermined industry structures (see appendix section A). Matrix $\mathbf{R}$ in fact can always be written as

$$\mathbf{R} = \mathbf{U} \cdot \mathbf{S} \cdot \mathbf{V}^T \qquad (24)$$

where $\mathbf{U}$ and $\mathbf{V}^T$ are two unitary matrices of convenient dimensions and $\mathbf{S}$ is a [11,11] diagonal matrix whose diagonal elements consist of the 11 scalars s_i. Scalars s_i are all positive and can be ordered in decreasing order. If we denote with $\mathbf{u}_i$ the columns of matrix $\mathbf{U}$ and with $\mathbf{v}_i$ the rows of matrix $\mathbf{V}^T$ we can express matrix $\mathbf{R}$ as

$$\mathbf{R} = \sum_i s_i \mathbf{u}_i \mathbf{v}_i \qquad (25)$$

each of the 11 elements of the summation represents a table composing [3].

If the demand impulse is chosen so that its structures is equal to, say, vector $\mathbf{v}_i$ all the elements of the summation, other than $\mathbf{s}_i \cdot \mathbf{u}_i \cdot \mathbf{v}_i$ would become equal to zero, since vectors $\mathbf{v}_i$ $(i = 1, \ldots, 11)$ are orthogonal, and matrix $\mathbf{R}$ would reduce to

$$\mathbf{R} = s_i \cdot \mathbf{u}_i \cdot \mathbf{v}_i \qquad (26)$$

Singular values $\mathbf{s}_i$, then, determine the aggregated effect of a final demand shock on output. For this reason we will call them macro multipliers. These macro multipliers are aggregated, in the sense that each

of them applies on all components of each macroeconomic variables taken into consideration, and are consistent with the multi-industry specification of the model[5].

We can than say that, given our matrix $\mathbf{R}$, we are able to isolate impacts of different (aggregate) magnitude, since that macro multiplier present in matrix $\mathbf{R}$, $\mathbf{s}_i$ can be activated through a shock along the demand structure $\mathbf{v}_i$ and its impact can be observed along the output structure $\mathbf{u}_i$.

Table 4: Macro-multipliers in $\mathbf{R}$

S_1	7.35
S_2	1.57
S_3	1.21
S_4	1.13
S_5	1,09
S_6	1.05
S_7	1.00
S_8	0.99
S_9	0.93
S_{10}	0.84
S_{11}	0.69

Table [4] shows the multipliers which are present in matrix $\mathbf{R}$. Macro multiplier 1 (7,35) is the dominating one for its order of magnitude. This means that a final demand vector change produces a change on the output vector 7.35 times greater. Macro multipliers from 2 to 6 amplify the effect of the shock, while the last four macro multipliers reduce it.

In the graphs in Figure [4(a)] we have reported, with the black histogram, the input structure $\mathbf{v}_1$ able to activate macro multiplier s_1, and, with the white histogram, the corresponding effect on industrial outputs given by $s_1 \mathbf{u}_1$. While in Figure [4(b)] and [4(c)] the same is done for $\mathbf{v}_2$, $s_2 \mathbf{u}_2$ and $\mathbf{v}_3$, $s_3 \mathbf{u}_3$.

The information emerging from Figures [4] can help in designing demand policies which are consistent with the observed structure of the interindustry interactions indicating the structure which are easiest to control. As an example let us refer to Table [5].

Table [5] has been built on the basis of matrix $\mathbf{R}$ multiplying it by a vector of final demand which has the same modulus of $\mathbf{f}$ in Table [3] but of composition equal to $\mathbf{v}_1$.

Each cell shows the growth of the ith industrial output, $\mathbf{x}_i$, caused by a demand impulse of f_j, as described in j^{th} row of the column $\mathbf{f}$ of

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[5]Given the problems connected with aggregation in multisectoral models, this feature of singular values s_i is not of minor relevance. They are aggregated multipliers consistently extracted from a multisectoral framework and their meaning holds both if we speak in aggregated or disaggregated terms.

Figure 4: Demand shocks structures and their impact on output composition
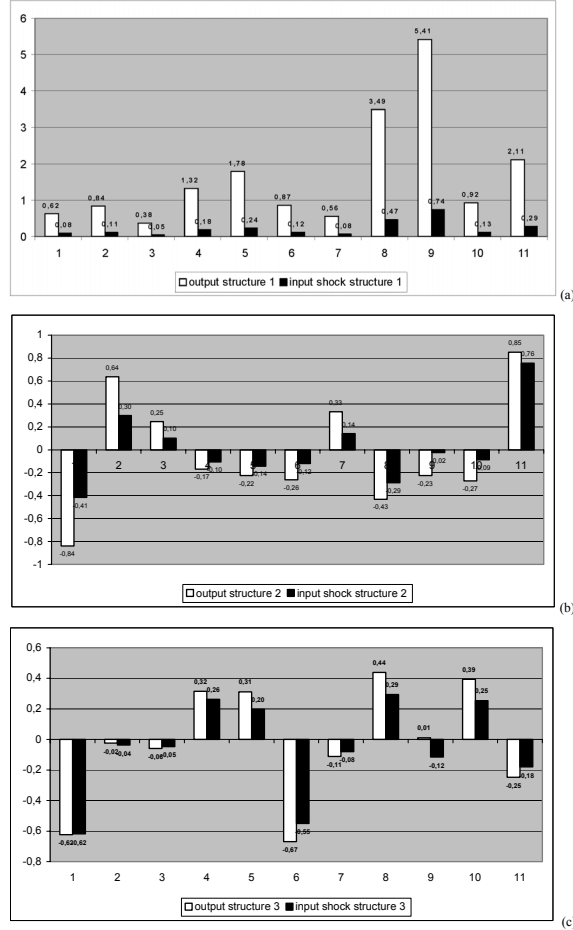
the table. The last four columns show, respectively, the total output vector change, $\mathbf{x}$, and the final demand shock, $\mathbf{f}$. Here two columns, $\mathbf{x}^2$ and $\mathbf{f}^2$, have been added in order to calculate the squares of the industry values of vector $\mathbf{x}$ and vector $\mathbf{f}$. This will facilitate the determination of the modulus of each vector, which will be done by taking the square root of the sum of each column. In fact when dealing with a vector representing a change it is convenient to refer to the commonly accepted measure of vector change which is the modulus of the vector. This is especially true if we want to take into consideration the possibility of considering also negative changes in some components of the vector.

From the last two values we can appreciate that the final demand

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Table 5: Direct and indirect effects on total output of final demand according structure 1

	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}	f_{11}	x	f	x^2	f^2
x_1	1.11	0.12	0.05	0.02	0.03	0.12	0.29	0.07	0.10	0.09	0.05	2.00	0.86	4.20	0.74
x_2	0.10	1.58	0.20	0.05	0.06	0.04	0.25	0.12	0.15	0.16	0.09	2.70	1.28	7.70	1.65
x_3	0.04	0.07	0.83	0.01	0.02	0.01	0.09	0.04	0.05	0.05	0.03	1.20	0.80	1.50	0.64
x_4	0.28	0.54	0.22	0.75	0.29	0.10	0.75	0.43	0.43	0.43	0.19	4.30	0.61	19.20	0.37
x_5	0.44	0.84	0.27	0.16	1.00	0.13	1.03	0.50	0.69	0.62	0.22	5.90	0.72	34.80	0.52
x_6	0.22	0.34	0.14	0.07	0.09	0.65	0.47	0.22	0.28	0.24	0.14	2.80	0.54	8.20	0.30
x_7	0.03	0.07	0.03	0.01	0.02	0.01	1.52	0.04	0.05	0.05	0.03	1.80	1.43	3.40	2.06
x_8	0.85	1.63	0.58	0.34	0.44	0.27	2.07	2.25	1.41	1.27	0.45	11.00	1.08	133.0	1.18
x_9	1.22	2.55	1.09	0.58	0.76	0.48	3.30	1.56	3.37	1.98	1.06	17.00	1.28	322.0	1.63
x_{10}	0.14	0.32	0.14	0.06	0.09	0.05	0.39	0.18	0.24	1.32	0.13	3.00	1.09	9.30	1.20
x_{11}	0.13	1.52	0.47	0.15	0.22	0.12	1.71	0.47	0.50	0.49	1.21	7.00	0.81	49.0	0.66
Sum	4.56	9.58	4.03	2.20	3.01	1.98	11.88	5.88	7.26	6.70	3.60	60.67	10.55	593.7	11

vector in Table [5] has the same module of that in Table [3] which are both equal to

$$mod(f) = \sqrt{\sum_i f_i^2} \qquad (27)$$

hence $\sqrt{11}$ =3,317. While the output vector module is equal to

$$mod(x) = \sqrt{\sum_i x_i^2} \qquad (28)$$

hence $\sqrt{593,7}$=24,36.

The ratio between the two modules is equal to mod(x)/mod(f) = 24,36/3,317 = 7,347 which is the value of multiplier s_1. This corresponds to a change in output of 60,674 which is higher than that obtained by chance in Table [3] that amounted to 58,975, and it is the highest performance the economy can attain. No higher performance can be attained.

Table [6] shows a similar application made with reference to the second multiplier s_2=1,5695. Here mod(x) = $\sqrt{27,10}$ = 5,2057, mod(f)=

Table 6: Direct and indirect effects on total output of final demand according structure 2

	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}	f_{11}	x	f	x^2	f^2
x_1	-2.28	0.13	0.03	-0.01	-0.02	-0.12	0.14	-0.06	-0.04	-0.04	0.12	-2.15	-1.78	4.65	3.15
x_2	-0.20	1.66	0.13	-0.03	-0.04	-0.04	0.12	-0.10	-0.06	-0.08	0.19	1.56	1.35	2.43	1.81
x_3	-0.08	0.07	0.54	-0.01	-0.01	-0.01	0.04	-0.03	-0.02	-0.03	0.06	0.55	0.52	0.27	0.27
x_4	-0.57	0.56	0.14	-0.43	-0.19	-0.10	0.36	-0.36	-0.16	-0.22	0.42	-0.54	-0.35	0.29	0.13
x_5	-0.91	0.88	0.18	-0.09	-0.65	-0.13	0.50	-0.42	-0.26	-0.33	0.48	-0.75	-0.47	0.56	0.22
x_6	-0.46	0.36	0.09	-0.04	-0.06	-0.66	0.23	-0.18	-0.11	-0.12	0.32	-0.62	-0.55	0.39	0.31
x_7	-0.06	0.07	0.02	-0.01	-0.01	-0.01	0.74	-0.03	-0.02	-0.02	0.06	0.73	0.70	0.53	0.49
x_8	-1.74	1.71	0.38	-0.20	-0.29	-0.28	1.01	-1.89	-0.52	-0.66	0.99	-1.49	-0.91	2.23	0.83
x_9	-2.52	2.67	0.70	-0.34	-0.50	-0.48	1.61	-1.31	-1.25	-1.03	2.33	-0.12	-0.48	0.01	0.23
x_{10}	-0.28	0.33	0.09	-0.04	-0.06	-0.05	0.19	-0.15	-0.09	-0.69	0.28	-0.45	-0.57	0.20	0.33
x_{11}	-0.27	1.60	0.30	-0.09	-0.14	-0.12	0.83	-0.40	-0.19	-0.26	2.67	3.93	1.80	15.48	3.24
Sum	-9.37	10.02	2.60	-1.28	-1.97	-2.00	5.79	-4.93	-2.70	-3.49	7.92	0.60	-0.75	27.10	11

$\sqrt{11}$ = 3,317 so that the module ratio is mod(x)/mod(f) = 1,5695.

Here, as we could expect from the results shown in Figure [4(b)] the shock slows down industries 1, 4, 5, 6, 8, 9 and 10 and expands industries 2, 3, 7 and 11. A shock with reverse sign would produce a reverse effect on the same industries. Revealing that the interactions in our economy create privileged sets of industries.

We have shown, through the use of numerical examples, that the parametric structure suggests the most effective demand policy, since whatever composition in the demand change other than v_1 causes less relevant results in terms of magnitude of the changes observed on industrial outputs. While the traditional Leontief multipliers analysis doesn't warrant that the results shown are the largest attainable.

6 Searching for a "summary" approach: The decomposition of the structural relationship between institutional sectors and industries

In this section we will explicitly consider the interaction between industries and institutional sectors operating on the structural matrices composing the loop in equations [13-22]. We will also utilize the singular value decomposition in the attempt of finding a "summary" measures of propagation (see section 2).

The interactions among industries and institutional sectors can be appreciated if one considers the direct and indirect effects of disposable incomes on industry outputs. From the extended income output circular flow we determine the structural matrix $\overline{\mathbf{R}}$ that links a unit change in disposable income by institutional sectors to total output by industries:

$$\overline{\mathbf{R}} = \mathbf{R} \cdot \mathbf{F} \tag{29}$$

where $\mathbf{F} = [\mathbf{F}^0 + \mathbf{K}]$ gives the link between disposable income and final demands shown in equation [18] and $\mathbf{R}$ is given in equation [23]. The loop disposable income total output will be given

$$\mathbf{x} = \overline{\mathbf{R}} \cdot \mathbf{y} \tag{30}$$

Its numerical determination is given in Table [7]. Two additional rows and columns show totals and quadratic moduli of the row (column).

We can perform the singular value decomposition of data in the table and determine the macro multipliers

Considering that matrix product $\overline{\mathbf{R}}^T \cdot \overline{\mathbf{R}}$ is the matrix of the deviations from zero of the effects of a unit shock and that the square roots of its eigenvalues are the singular values of matrix $\overline{\mathbf{R}}$, we can

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Table 7: Direct and indirect effects of disposable incomes on industry outputs

	I	II	III	IV	V	VI	VII	Totals	Moduli
x_1	0.13	0.11	0.10	0.10	0.10	0.09	0.06	0.71	0.27
x_2	0.15	0.15	0.14	0.14	0.15	0.14	0.11	0.97	0.37
x_3	0.08	0.07	0.06	0.06	0.06	0.05	0.04	0.41	0.16
x_4	0.29	0.36	0.40	0.45	0.51	0.61	0.23	2.85	1.12
x_5	0.26	0.46	0.61	0.77	0.98	1.33	0.27	4.67	2.01
x_6	0.34	0.31	0.28	0.28	0.28	0.25	0.18	1.93	0.74
x_7	0.06	0.06	0.05	0.05	0.05	0.04	0.03	0.35	0.13
x_8	0.83	1.07	1.23	1.45	1.73	2.16	0.55	9.01	3.66
x_9	2.04	1.95	1.86	1.92	1.98	1.95	1.29	13.00	4.95
x_{10}	0.30	0.27	0.25	0.25	0.25	0.22	0.16	1.68	0.65
x_{11}	0.40	0.42	0.43	0.47	0.52	0.58	1.49	4.29	1.88
Totals	4.86	5.23	5.43	5.95	6.61	7.43	4.41		
Moduli	2.33	2.39	2.43	2.64	2.93	3.33	2.09		

Table 8: Macro multipliers and cumulative percent sum

	Macro multiplier	Cumulative percent sum
1	6.73	74%
2	1.34	89%
3	0.98	100%
4	0.02	100%
5	0.00	100%
6	0.00	100%
7	0.00	100%

conclude that each singular value in Table [8] can be interpreted as the share of the deviations related to the associated eigenvectors. If we determine the cumulated percentage shares, we see that the first two singular values cover the 89 per cent of total deviations. This means that we can confine our analysis of intersectoral and interindustry interactions to the first two macro multipliers to get results valid for the 89 per cent of the cases. Rather than considering matrix $\overline{\overline{\mathbf{R}}}$, which can be decomposed into the sum of seven "impact" components each one determined by a macro multiplier:

$$\mathbf{R}^0 = s_1 \cdot \mathbf{u}_1 \cdot \mathbf{v}_1 + s_2 \cdot \mathbf{u}_2 \cdot \mathbf{v}_2 + \ldots + s_7 \cdot \mathbf{u}_7 \cdot \mathbf{v}_7 \qquad (31)$$

we can refer to matrix

$$\mathbf{R}^0 = s_1 \cdot \mathbf{u}_1 \cdot \mathbf{v}_1 + s_2 \cdot \mathbf{u}_2 \cdot \mathbf{v}_2 \qquad (32)$$

where components greater than 2 have been neglected with the aim of obtaining "summary" measures. Now the economic interactions are completely determined by the first two aggregated macro-multipliers s_1 and s_2.

We note that in matrix $\mathbf{R}^0$, vectors

$$s_1 \cdot \mathbf{u}_1 = \begin{bmatrix} s_1 u_{1,1} \\ s_1 u_{2,1} \\ s_1 u_{3,1} \\ . \\ . \\ . \\ s_1 u_{11,1} \end{bmatrix}, \qquad s_2 \cdot \mathbf{u}_2 = \begin{bmatrix} s_2 u_{1,2} \\ s_2 u_{2,2} \\ s_2 u_{3,2} \\ . \\ . \\ . \\ s_2 u_{11,2} \end{bmatrix} \qquad (33)$$

are the result of splitting the two macro multipliers into the eleven output sector. These two vectors represent both how each of the macro multipliers affects outputs and how each industry output is affected by the two macro multipliers, which quantify the magnitude of industry-sector interactions: As we stressed in section 2, the aim

Table 9: Forward dispersion i.e. impacts on industry outputs of intersectoral interactions, in terms of macro multipliers

	First impact component $\mathbf{u}_1 \cdot s_1$	Second impact component $\mathbf{u}_2 \cdot s_2$	Forward Dispersion (Modules)	Percent forward dispersion
x_1	0.27	0.03	0.27	1.7
x_2	0.37	0.04	0.37	2.4
x_3	0.15	0.02	0.15	1.0
x_4	1.12	-0.11	1.12	7.1
x_5	1.90	-0.55	1.98	12.6
x_6	0.72	0.08	0.73	4.6
x_7	0.13	0.02	0.13	0.8
x_8	3.59	-0.67	3.65	23.2
x_9	4.91	0.42	4.93	31.3
x_{10}	0.63	0.08	0.64	4.1
x_{11}	1.50	0.92	1.76	11.2
Modules	6.73	1.34		

of the *sensitivity of dispersion* in the Rasmussen definition, τ_i, -which generated the concept of the forward linkage- measures the extent to which industries draw upon industry i and the degree of relevance of each industry as a supplier.

As we see from Table [9], the expansion of the ith industry output is quantified by vector $[s_1 \ u_{1i}, \ s_2 \ u_{2i}]$ and its module. It is to be noted that the industry expansion effect is measured with reference to the two macro multipliers independently from the fact that such multipliers have been activated by a change in final demand or a change in disposable incomes influencing final demands. This feature allows for a generalization of the sensitivity of dispersion concept. This concept can be used both in the case that the model is limited to the Leontief inverse and to case were a larger output/income model is used that

includes also the income distribution process. In order to avoid misinterpretation we will define the forward dispersion, fd_i, as the change in the value of the sales by industry i (to face a demand vector generated by an increase in disposable income in all sectors). The percent forward dispersion can be easily obtained dividing forward dispersion by its total value.

Table [9] produces an ordering of industries according the forward of dispersion: Industry 9 Transport and Trade (31,3%), 8 Manufacturing (23,2%), 5 Machinery and Cars (12,6%), 11 Service non market (11,2%), 4 Metal & chem. Products (7,1%), 6 Food (4,6%), 10 Service market (4,1%), 2 Oil (2,4%), 1 Agriculture, (1,7%), 3 Energy (1,0%), 7 Tobacco and Alcoholic Beverages (0,8%).

On the other hand vectors

$$s_1 \cdot \mathbf{v}_1 = [s_1 \cdot v_{1,1}, \ldots, s_1 \cdot v_{1,7}] \quad s_2 \cdot \mathbf{v}_2 = [s_2 \cdot v_{2,1}, \ldots, s_2 \cdot v_{2,7}] \quad (34)$$

split the same two macro multipliers into the seven institutional sectors and represent how the change in sectoral disposable income influences the two macro-multipliers.

Again from section 2, the aim of index π_j, the power of dispersion in the Rasmussen definition -which generated the concept of backward linkage- was that of measuring the extent to which an increase in final demand for products of industry j is dispersed throughout the system of industries.

If we introduce in the interindustry model, institutional sectors and income distribution, final demand will no more be exogenous but explained by income distribution. Whatever multisectoral macro variable will it be, the index will quantify the degree of relevance of each component of such macro variable in stimulating the multipliers.

If the model under analysis had been the loop between final demand and output vectors [34] would have well represented the backward linkage i.e. the expansion caused by an expansion in industry j. By analogy we can define backward dispersion, bd_j, as the change in the value of the purchases by those industries that produce goods according the consumption patterns of income sector j. Backward dispersion can be also determined in percent terms as in Table [10].

We note that the fourth column of Table [9] corresponds to the modules of the rows of table $\mathbf{R}^0$ and that the same column in Table [10] gives the modules of the columns of table $\mathbf{R}^0$, which at his turn approximates to be $\overline{\mathbf{R}}$.

We can give a graphical representation of each element in the four vectors. We will define the axis of the first macro multiplier, on which we measure the elements of vectors $s_1\mathbf{u}_1$, $s_1\mathbf{v}_1$ and the axes of the second macro multiplier, where we measure the elements of vectors $s_2\mathbf{u}_2$,

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Table 10: Backward dispersion i.e. impacts of a unit disposable income shock on economic interactions, in terms of macro multipliers

	First impact component $v_1 \cdot s_1$	Second impact component $v_2 \cdot s_2$	Backward Dispersion (Modules)	Percent backward dispersion
I-Income class households	2.22	0.41	2.26	12.6
II-Income class households	2.35	0.19	2.36	13.2
III-Income class households	2.42	0.02	2.42	13.5
IV-Income class households	2.64	-0.11	2.64	14.7
V-Income class households	2.91	-0.3	2.93	16.3
VI-Firms	3.24	-0.63	3.3	18.4
VII-Administration	1.73	1.05	2.02	11.3
Modules	6.73	1.34		

$s_2\mathbf{v}_2$. Then we will represent the couples $(s_1\mathbf{v}_{1,i},\ s_2\mathbf{v}_{1,i})$ i=1,…,7, with seven arrows, showing how the change in disposable income impacts on intersectoral interactions (backward dispersion), in terms of the two macro multipliers; and couples $(s_1\mathbf{u}_{1,i},\ s_2\mathbf{u}_{1,i})$ i=1,…,11, with eleven dots, showing how intersectoral interactions impact on industry outputs (forward dispersion).

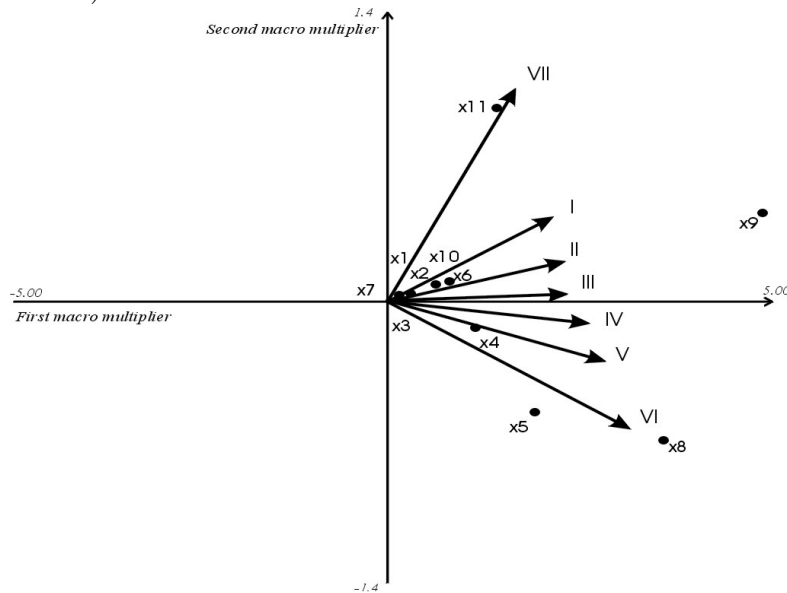Figure 5: Sector and industry interactions -Backward and forward dispersions (absolute levels)

Figure [5] shows that, in addition to the information based on the modules of the vectors, some further information can be achieved referring to the directions of each vector. In order to perform consistent

comparisons, independently from the unit measures effects of outputs and incomes, we need to standardize data in Table [7] taking the deviations from the mean values and dividing by the standard deviations. We note that the singular value decomposition of standardized data will result in the eigenvalue decomposition of matrices $\mathbf{R}^T\mathbf{R}$ and $\mathbf{R}\mathbf{R}^T$ which represent the correlation matrices of sectoral incomes and industry outputs respectively. We will then get the diagram in Figure [6]:

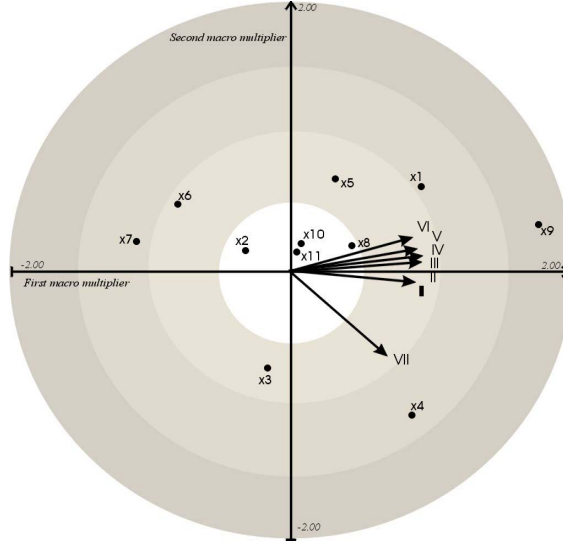Figure 6: Sector and industry interactions -Backward and forward dispersions standardized

Figure [6] allows for the identification of clusters of industries that move together, i.e. respond linearly, to intersectoral interactions as quantified by the two macro multipliers. This is done considering that the angular distance of two dots will represent the correlation coefficient since:

$$Corr(\mathbf{x}_i, \mathbf{x}_j) = \cos\beta = \frac{\mathbf{x}_i \cdot \mathbf{x}_j}{\mid \mathbf{x}_i \mid \cdot \mid \mathbf{x}_j \mid}$$

in fact, if two industries "move together", we have to expect that they will be located on the same line, relative to the two macro macro multipliers. From data in Figure [6], a correlation table is derived as shown in Table [11]. For correlation coefficients greater than 90 per cent we can identify a set of six industries clusters:

1st cluster: Positive correlation characterizes industry 1 Agriculture with respect to 8 Manufacturing and 9 Transport and Trade; 2nd cluster: Positive correlation between industry 2 Oil and 6 Food, 7

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Table 11: Correlation coefficients between industries

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
x_1	1.00										
x_2	-0.57	1.00									
x_3	-0.72	-0.16	1.00								
x_4	0.10	-0.88	0.62	1.00							
x_5	0.85	-0.06	-0.98	-0.43	1.00						
x_6	-0.45	0.99	-0.30	-0.93	0.09	1.00					
x_7	-0.73	0.98	0.05	-0.75	-0.27	0.94	1.00				
x_8	0.99	-0.7	-0.59	0.27	0.75	-0.59	-0.83	1.00			
x_9	0.92	-0.84	-0.39	0.48	0.58	-0.76	-0.94	0.97	1.00		
x_{10}	0.78	0.06	-1.00	-0.54	0.99	0.20	-0.15	0.67	0.48	1.00	
x_{11}	0.66	0.24	-1.00	-0.68	0.95	0.38	0.03	0.52	0.31	0.98	1.00

Tobacco & Alcoholic Beverages: 3rd cluster: Negative correlation is observed between industry 3 Energy, and 5 Machinery and Cars, 10 Service market, 11 Service non market; 4th cluster: Negative correlation between industry 4 Metal and chemical Products and 6 Food; 5th cluster: Negative correlation between industry 7 Tobacco and Alcoholic Beverages and 9 Transport and Trade; 6th cluster: Negative correlation between industry 7 Tobacco and Alcoholic Beverages, 9 Transport and Trade. Other four clusters are implied by the previous as x_5, x_{10} and x_{11}; x_6 and x_7; x_8 con x_9, x_{10} and x_{11}.

For what concerns the backward dispersion, the modulus of each vector labelled I, II, III, IV, V, VI, VII, represent the stimulus forwarded to the interindustry interactions by a unit change in disposable income by institutional sector. From Figure [6] we note that in our example the effects of disposable incomes of institutional sectors from I to VI are highly correlated, more than 90 per cent in terms of correlation coefficient. Only sector VII, Administration, seems to exhibit a different pattern. Its correlation with the other sectors is of decreasing order from 80 per cent with sector I to 53 per cent with sector VI.

Figure [6], in addition, allows for a cross comparison sectors/industries which can identify the "strength" of the link between sectors and industries in terms of cross correlation coefficients.

Table [12] shows high positive correlations between sector I and industry 9; sector II and industries 8 and 9; sector III and industries 8 and 9; sector IV and industries 8 and 9; sector V and industries 1, 8 and 9; sector VI and industries 1, 8 and 9; sector VII and industries 4.

While high negative correlations are observed between sector I and industries 6 and 7; sector II and industries 2 and 7; sector III and industries 2 and 7; sector IV and industry 7; sector V and industry 7; sector VI and industry 7 Tobacco and Alcoholic Beverages; sector VII and industries 2 and 6.

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Table 12: Cross Correlation coefficients between industries and sectors

	I	II	III	IV	V	VI	VII
1	0.78	0.83	0.87	0.89	0.92	0.95	0.24
2	-0.96	-0.93	-0.90	-0.88	-0.85	-0.80	-0.93
3	-0.12	-0.22	-0.28	-0.33	-0.38	-0.46	0.50
4	0.70	0.63	0.58	0.54	0.49	0.41	0.99
5	0.33	0.42	0.48	0.53	0.57	0.65	-0.31
6	-0.91	-0.87	-0.83	-0.80	-0.77	-0.71	-0.97
7	-1.00	-0.99	-0.97	-0.96	-0.94	-0.91	-0.84
8	0.87	0.92	0.94	0.96	0.97	0.99	0.40
9	0.96	0.98	0.99	1.00	1.00	1.00	0.60
10	0.22	0.31	0.38	0.42	0.47	0.55	-0.42
11	0.04	0.13	0.20	0.25	0.30	0.39	-0.58

Among sectoral and industrial interactions these emerge as the strongest. Here it seems that sectoral disposable income has a direct influence on industrial output. In these cases the structure of backward dispersion is equal to that of forward dispersion since the sectoral disposable-income change activates the macro multipliers in the same combination in which industrial outputs are stimulated.

7 Conclusions

The origin of linkage analysis, in the study of propagation phenomena through industries, was that of finding "summary" measures of dispersion and of applying them on interindustry data in a statistical way. However, in later developments, the original statistical approach has been progressively abandoned and the interpretation of these measures have definitely become deterministic. Further developments have proposed problem specifications, such that based on the assumption of constant market shares, conflicting with the hypothesis of fixed technical coefficients. On the other hand developments in national accounts have provided a consistent data base for the enlargement of the traditional Leontief framework to problems of income distribution on the lines explored by Miyazawa.

Our attempt has been that of taking inspiration from some of these developments to design measures of dispersion, either "summary" and "statistical", that can be applied both to a traditional Leontief framework and to an enlarged model, where income distribution can be also taken into consideration. The results have been discussed on the basis of a specific regional model whose data base we have tried to render consistent, having in mind a social accounting scheme.

The emerging enlarged income flow has been analyzed identifying the macro multipliers that "rule" the flow. Once identified these multipliers, that represent the potential scale of all the possible ty-

pes of dispersions through industries and sectors, we evaluated both backward and forward dispersions with reference to them. This procedure generates a set of indices -in absolute and percent values- for the industry-forward-dispersion and sector-backward-dispersion which quantify, respectively, the change in the value of the sales by industry i to face a demand vector generated by an increase in disposable income in all sectors, and the change in the value of the purchases by those industries that produce goods according the consumption patterns of income sector j.

An extension of the method has also been provided in terms of a "summary" graphical representation. The standardization of data, in fact, produces a representation, explainable in terms of correlation analysis, which allows for an immediate interpretation of the strength of the mutual links among and between the disaggregated components of total output and disposable income. A synthetic picture of the working of sector-industry-interactions is then attained in graphical and quantitative terms.

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A Spectral Decomposition

The decomposition proposed can be applied both to square and to non-square matrices. Here the general case of non-square matrix $\overline{\mathbf{R}}$ will be shown. The square matrix case is easily developed along the same lines.

Let us consider matrix $\mathbf{W}$, the square of our [11,7] structural matrix $\overline{\mathbf{R}}$:

$$\mathbf{W} = \overline{\mathbf{R}}^T \cdot \overline{\mathbf{R}}$$

Matrix $\mathbf{W}$ has a positive definite or semi definite square root. Given that $\mathbf{W} \geq 0$ by construction, its eigenvalues (λ_i) $i = 1, \ldots, 7$ shall be all real non negative (Lancaster and Tiesmenetsky, 1985).

The non zero eigenvalues of matrices $\overline{\mathbf{R}}^T \overline{\mathbf{R}}$ and $\overline{\mathbf{R}}\overline{\mathbf{R}}^T$ coincide. The system of eigenvectors $[\mathbf{u}_i \ i=1,....,11]$ for $\overline{\mathbf{R}}^T \overline{\mathbf{R}}$ and $[\mathbf{v}_i \ i=1,....,7]$ for $\overline{\mathbf{R}} \overline{\mathbf{R}}^T$ are orthonormal bases.
We get then

$$\overline{\mathbf{R}}^T \cdot \mathbf{u}_i = \sqrt{\lambda_i} \cdot \mathbf{v}_i \qquad i = 1, \ldots, 7$$

and

$$\overline{\mathbf{R}}^T \cdot \mathbf{u}_i = 0 \qquad 8 \leq i \leq 11$$

We can construct the two matrices

$$\mathbf{U} = [\mathbf{u}_1, \mathbf{u}_2, \cdots, \mathbf{u}_{11}] \qquad and \qquad \mathbf{V} = [\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_7]$$

Since by definition $s_i = \mathbf{v}i$ with i=1,....,11 we get

$$\overline{\mathbf{R}}^T \cdot \mathbf{U} = [s_1 \cdot \mathbf{v}_1, s_2 \cdot \mathbf{v}_2, \ldots, s_7 \cdot \mathbf{v}_7, 0, \ldots\ldots, 0] = \mathbf{V} \cdot \mathbf{S}$$

Strucutral matrix $\overline{\mathbf{R}}$ in equation [30] can be then decomposed as

$$\mathbf{x} = \mathbf{U} \cdot \mathbf{S} \cdot \mathbf{V}^T \cdot \mathbf{y} \tag{35}$$

$\mathbf{V}$ is an [7,7] unitary matrix whose columns define the 7 reference structures for disposable income:

$$\mathbf{v}_1 = \begin{bmatrix} v_{1,1} & v_{1,2} & v_{1,3} & . & . & . & v_{1,7} \end{bmatrix}$$

$$\mathbf{v}_2 = \begin{bmatrix} v_{2,1} & v_{2,2} & v_{2,3} & . & . & . & v_{2,7} \end{bmatrix}$$

$$\cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots$$

$$\mathbf{v}_7 = \begin{bmatrix} v_{7,1} & v_{7,2} & v_{7,3} & . & . & . & v_{7,7} \end{bmatrix}$$

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U is an [11,11] unitary matrix whose columns define 11 reference structures for output:

$$\mathbf{u}_1 = \begin{bmatrix} u_{1,1} \\ u_{2,1} \\ u_{3,1} \\ . \\ u_{11,1} \end{bmatrix}, \mathbf{u}_2 = \begin{bmatrix} u_{1,2} \\ u_{2,2} \\ u_{3,2} \\ . \\ u_{11,2} \end{bmatrix}, \ldots, \mathbf{u}_{11} = \begin{bmatrix} u_{1,11} \\ u_{2,11} \\ u_{3,11} \\ . \\ u_{11,11} \end{bmatrix}$$

and **S** is an [7,7] diagonal matrix of the type:

$$\mathbf{S} = \begin{bmatrix} \mathbf{s}_1 & 0 & 0 & . & 0 \\ 0 & \mathbf{s}_2 & 0 & . & 0 \\ 0 & 0 & \mathbf{s}_3 & . & 0 \\ . & . & . & . & . \\ 0 & 0 & 0 & . & \mathbf{s}_7 \end{bmatrix}$$

Scalars s_i are all real and positive and can be ordered as $s_1 > s_2 > \ldots > s_7$. Now we have all the elements to show how this decomposition correctly represents the macro multipliers that quantify the aggregate scale effects and the associated structures of the impact of a shock in disposable income on total output.

In fact if we express the actual vector **y** in terms of the structures identified by matrix **W**, we obtain income demand vector, $\mathbf{y}^0$, expressed in terms of the structures suggested by the $\overline{\overline{\mathbf{R}}}$:

$$\mathbf{y}^0 = \mathbf{V} \cdot \mathbf{y} \tag{36}$$

On the other hand we can also express total output according the output structures implied by matrix $\overline{\overline{\mathbf{R}}}$:

$$\mathbf{x}^0 = \mathbf{U}^T \cdot \mathbf{x} \tag{37}$$

Equation [35] then becomes through equations [36] and [37]:

$$\mathbf{x}^0 = \mathbf{S} \cdot \mathbf{y}^0 \tag{38}$$

which implies:

$$x_i^0 = s_i \cdot y_i^0 \tag{39}$$

where i=1,...,7. We note that matrix $\overline{\overline{\mathbf{R}}}$ hides seven fundamental combination of the outputs. Each of them is obtain multiplying the corresponding combination of incomes by a predetermined scalar which has in fact the role of aggregated macro multiplier.

The complex effect on the output vector of income shocks can be reduced to a multiplication by a constant s_j.

The structures we have identified play a fundamental role in determining the potential behavior of the economic system, i.e. the behavior of the system under all possible shocks. We can in fact evaluate which will be the effect on output of all income possible structures. This is easily done imposing in equation [35] a vector whose modulus is constant, say equal to one, but whose structure can assume all possible configurations. If vector **y** in equation [35] is such that
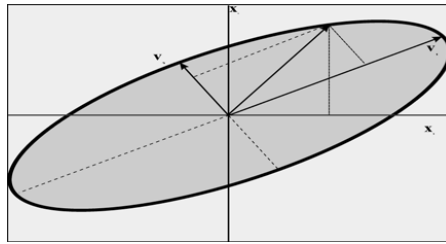
$$\sqrt{\sum_j y_j^2} = 1 \tag{40}$$

then geometrically we mean that the income vector describes a sphere of unit radius: the unit ball.It rotates around the origin, as in Figure [7(a)], assuming all the possible structures, including those implied by the columns of matrix **V**. Correspondingly the vector of total output will describe an ellipsoid with semi-axes of length $s_1, \ldots, s_7$, oriented according the directions designated by the columns of matrix **U**, as in figure [7(b)]. This ellipsoid is sometimes called the isocost of income control.

When income vector crosses a structure in **V**, the vector of total output crosses the corresponding structure in **U** and the ratio between the moduli of the two vectors is given by the corresponding scalar s.

Figure 7: Unit ball and corresponding elipsoid for disposable income
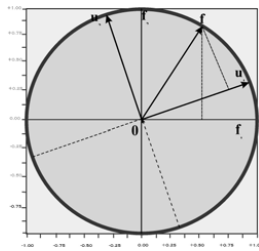
(a) Unit ball for disposable income   (b)Corresponding elipsoid

B   Social Accounting Matrix data

The total output data base comes directly from the regional multisectoral accounting which is articulated into 11 I-O branches. Flows are given by the amount of intermediate goods ($B^{M,M}$) used in production process evaluated *factory-gate*.

The availability of a national table of intersectoral flows for the same period (*TEI96*) helps in the acquisition of information on intermediate consumptions and final demand components. In our case we do not have a table elaborated by ISTAT for *Marche* but a table constructed by IRPET within a bi-regional structure. *TEI* has been used as data base for reconstruction non available information following a methodology proposed in (Miller and Blair, 1985). Operations on the first block refer to the estimation of imports of intermediate and final goods by origin industry of region rest of Italy and industries and institutional sectors of region Marche as users. The estimation has been performed using import coefficients from bi-regional *SAM* 1995 North-Center and South-Isles. Flows are net from imports from region *rI*.

Table 13: Intermediate consumption (billions of Italian 1996 liras)

	1	2	3	4	5	6	7	8	9	10	11
Agriculture	626.8	0	0	11	3	1214	155	222	169	4	66
Oil	61.2	955	321	272	43	60	16	261	402	92	198
Energy	34	11	99	122	34	35	7	159	171	36	86
Metal & chem. Products	227.7	17	46	3131	2001	120	103	4066	227	174	289
Machinery and Cars	40.6	1	39	115	1596	1438	4	946	253	7	704
Food	460.2	0	0	738	0	811	52	894	3379	12	168
Tobacco & Alc. Beverages	0.6	0	0	1	0	5	2	1	803	0	22
Manufacturing	37.6	12	70	600	590	142	77	7504	1771	130	665
Trasp & Trade	433.7	48	83	1656	1608	649	185	3501	7018	681	1041
Service market	0.1	1	13	29	629	1	2	8	14	0	0
Service non market	0	0	0	0	0	0	0	0	0	0	0

The production block closes with endogenous final demand (households consumption). The information comes from *TEI* and regional accounts articulated into 11 branches.

The disaggregation of consumption according five income classes has been performed using data on consumption by the Bank of Italy (Indagine sul Bilancio delle Famiglie (*BF*) 1995, (Banca d'Italia, 1995). We needed to build a bridge matrix linking durables and non durables with I-O consumption by households.

In the value added generation block value added is articulated into the two components which are present in the original table: value added at factors' cost, Va, and net indirect taxes (*I.I.n*). A greater detail is reached through data of the regional accounts: Wages-Salaries and Other Incomes (*AR*). The residual value added at factors' cost gives the Gross Operating Surplus ($va_{2,j}$) producing the table:

Table 14: Final domestic Consumption of Households Income Class

	I_Income class	II_Income class	III_Income class	IV_Income class	V_Income class
1	54	116	330	205	17
2	75	160	456	284	23
3	74	158	449	279	23
4	75	221	944	469	23
5	31	247	1652	672	8
6	262	561	1598	993	80
7	51	109	309	192	16
8	300	879	3757	1867	90
9	1209	2588	7370	4581	371
10	255	546	1555	966	78
11	54	116	330	205	17

$$Va^{IO} = \left[\begin{array}{cccc} va_{1,1} & \cdot \quad \cdot \quad \cdot & va_{1,m} \\ va_{2,1} & \cdot \quad \cdot \quad \cdot & va_{2,m} \end{array} \right] \tag{41}$$

This further component gives an aggregate which changes in its composition according the industry: it represents in fact: profits, capital incomes and income independent, with capital consumptions.

Table 15: Value Added by industry sectors

	1	2	3	4	5	6	7	8	9	10	11
Va_1	626.8	0	0	11	3	1214	155	222	169	4	66
Va_2	61.2	955	321	272	43	60	16	261	402	92	98

The third component that exhausts the allocation of value added to components is Indirect taxation which has been conveniently introduced in Table 15. Value added by components is then attributed to the factors' owners. The block completes the primary distribution process and shows the amount of value added components to be attributed to institutional sectors. Wages and Salaries will represent an exclusive inflow of households, we need to make here the assumption that Employers' social on are attributed to households. This fact would imply a further assumption of outflows from households in the secondary distribution towards public administration and firms. The accounting of institutional sectors and sub-sectors needs the identification of the sectoral inflows. The ownership of the factors represents the main selection criterion. A new table is then constructed for value added by institutional sectors (Va^{SI}), where rows a represent institutional sectors ($i=1,..,s$) and columns va components (factors):

$$Va^{SI} = \begin{bmatrix} va_{1,1} & va_{1,2} \\ . & . \\ . & . \\ . & . \\ va_{s,1} & va_{s,2} \end{bmatrix} \tag{42}$$

In this table the second component, Other Incomes, presents some difficulties deriving from the fact that we need to refer to accounting for macro regions in order to determine how Mixed Incomes, that appears in regional accounting, can be splitted into the share due to Public Administration and in that flowing to households and individual firms.

Once subtracted the Operating Surplus gross (AR^{PA}), the rest is splitted on the basis of two coefficients:

$$\sum_j va_{2,j} - AR^{PA} = AR \tag{43}$$

The allocation of the residual to the sectors is given by:

$$AR^F = g \cdot AR \quad and \quad AR^I = (1 - g) \cdot AR \tag{44}$$

where g is a function of the structure of small and medium size firms in the region, having in mind the structure of the distribution of the Operating Surplus gross to the whole economy's sectors. Given the amount AR^F it was possible to disaggregate the flow through the inflows structure in BF, for the 5 *Households Income classes*.

Table 16: Primary distribution of Income by institutional sectors

	Va_1	Va_2
I_Income class	133	529
II_ Income class	2006	875
III_Income class	10379	5303
IV_Income class	6998	6244
V_Income class	364	1451
Firms	0	12128

The allocation of this aggregate among institutional sectors requires further information on capital incomes in order to determine primary incomes balance and then the gross regional income. The operating surplus gross in this phase is correct by capital income flows between institutional sectors. Hence it is needed to find the interests, dividends, surplus, insurance policy holders and rents. A single flow needs of the sectoral and sub sectoral institutional accounting regional, but without the local accounting imposes the to use the new criterion

by the imputation. It is bases on framework to the capital income distribution between institutional sectors at macro regional level, taking that the survey data, when are disposable, must be applied, also it is to non official font and relative to the single aggregated.

The capital income to the domestic sectors (**K**) can be represented by Gross Operating Surplus and the structure of the matrix **B** [3,3] (Households, Firm and out of the quadrant Public Administration):

$$\mathbf{K} = \mathbf{B} \cdot AR \qquad (45)$$

from which we can determine the value of the capital income by *Households Income Class*, applying the *BF* framework.

The standard economic aggregated (*Gross Regional Income and Gross Domestic Product*) is not still to determinable because it needs to insert the outside flows of the quadrant (Value Added, capital income and Indirect Tax), that we describe at the next part. The quadrant finish with the block of the secondary income distribution, with all sectoral transfers that doesn't increase the GDP. We record in this block the flow that are added to others just insert above when we build the income distribution to owner of primary factors. The determination of domestic flows to be attributed to various cells is based on the structure that can be determined from *BF* and from its integration with the regional social accounting matrix. The determination of the redistribution accounts needs the reconstruction of a set of sub matrices which regard the items accounted for in secondary distribution accounts. In this way we insert all available information that regional accounts do not provide.

In the block the flows regarding public administration are much easier to be determined with respect to transfers internal to institutional sectors. The aggregation of sub blocks leads to the determination of the matrix of transfers: row and column related to households are disaggregated by sub sectors with a framework given to the *BF*. No information to the side of entries induces us, in this working progress, to attribute this flows to the relative sub sectors, under assumption of balance spending, because we know the gross saving.

Table 17: Secondary distribution of Income among institutional sectors

	I_Income class	II_Income class	III_Income class	IV_Income class	V_Income class	Firms
I_Income class	42	0	0	0	0	415
II_ Income class	0	4	0	0	0	1375
III_Income class	0	0	17	0	0	3988
IV_Income class	0	0	0	3	0	3309
V_Income class	0	0	0	0	0	251
Firms	41	160	553	380	12	3637

Section I of the SAM, through Tables [13 - 17], has been disaggregated to allow the introduction of institutional sub-sectors.

We now refer to the flows among domestic and non domestic operators. The first case is given by the regional in and outflows which involve Public Administration. The vast majority of the in flows in this block are all type of taxes either direct and indirect, contributes, unilateral transfers and social security. As to outflows subsidies, social security and transfers to households and firms, interests on public debt. Indirect taxes on output come from industrial accounting where we find Indirect taxes on products, on imports and *VAT*.

Table 18: Government Indirect and Import Tax

	1	2	3	4	5	6	7	8	9	10	11
Indirect Tax	45	75	1	26	149	32	20	111	0	0	0
Import Tax	-309	1630	73	66	120	77	876	2048	1569	157	0

From primary distribution of income, other than the value added share we have already described, we have also a residual Gross Operating Surplus flow to be referred to the activity developed by Public Administration within the region. The primary distribution block develops with the flows related to capital incomes by other institutional sectors determined with reference to the consolidated account of Public Administration.

The secondary distribution block shows the re distributive effect of transfers and is determined in connection with the construction of the sub matrices of income account integrated with information from *BF*. In particular, as to inflows, we register direct taxes on income and wealth (Irpef, Iperg) Social contributions and other current taxes.

The Irpef inflow has been disaggregated by income classes on the basis of the structure given in *BF* through elaboration of individual data for Marche according the fiscal legislation 1996. Within each class has been determined an average income, on which the progressive taxation and deductions could be applied by income classes.

The tax structure has been then applied to the Irpef inflow as indicated by the Income Agency (Ministry of Finance) for Marche. The same procedure has been utilized for Ilor (local tax on incomes) considering that in 1996 this tax was applied only on other incomes considering consistent deductions.

The amount of benefits outflows was determined as an aggregated amount from regional accounts (ISTAT, 1996) and other information (*Inps*), while the decomposition by income classes was done according the amount of average deductions. The Irpeg inflow of Marche has been attributed to Firms together with the Ilor residual.

Table 19: Government flows from others institutional sectors

	I_Income class	II_Income class	III_Income class	IV_Income class	V_Income class	Firms
Government	203	1085	5344	4042	408	2585
Current Tax on Income	161	475	2205	1925	299	2242
Social transfers	40	604	3126	2108	110	0

In column of section V we register collective consumption expenditures, as IO final demand component (*TEI*), interest transfers from Public Administration to households, social security benefits, and direct transfers to corporations. (Inps and *BF*).

Table 20: Government transfers

	Public Administration
I_Income class	1272
II_ Income class	2534
III_Income class	6653
IV_Income class	3400
V_Income class	606
Firms	5720

Section VI shows one row and column which illustrate the inflows and outflows of the rest of the World the output block imports and exports that are given in *TEI*; In the primary income distribution the net value added flows and capital incomes determined by the construction of the blocks in section **I**; for secondary distribution block we get all direct transfers form and towards the rest of the world.

We have concentrated on sections **I**, **V** and **VI**, but a similar procedure has been adopted for determining also sections **II**, **III** and **IV** in order to obtain the bi regional Social Accounting Matrix, whose scheme is shown in Table 1.