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Uncertainty, Trade Integration
and the Optimal Level of Protection
in a Ricardian Model with a Continuum of Goods

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Abstract

This paper analyzes how increasing trade integration affects individual utility when the international specialization pattern is stochastic, i.e. when the number of varieties each country produces depends on the realization of a random variable. I employ a Ricardian continuum of goods model to show that in this case a trade off emerges. As in the standard model, higher trade integration reduces prices and increases expected real income. However, higher trade integration, reducing the number of active sectors in the economy, also increases the displacement cost the worker would suffer in a bad state (i.e. when the sector she is employed into has to close down because, ex-post, the foreign country's competing sector results to be more efficient). The main result of the model is that there exists an optimal level of protection that it is higher the smaller the price reduction induced by trade integration and the more technologically similar are countries.

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1 Introduction

The process of globalization of production that has taken place in the last two decades has been accompanied by increasing concern about its economic and social effects. Even if by now thousands of theoretical and empirical papers have analyzed the many issues involved, the debate on the economic benefits and cost of increasing trade integration is still open. Limiting ourself to the discussion about its the static effects, there are two main positions in the profession. On one side, there are the ones that emphasize the large gain in allocational efficiency that would result from free international exchange. On the contrary, others point to a series of possible negative consequences of increasing trade openness, among which the most important are higher income inequality and income risk. But, while the literature on the allocation and distributional effects of trade is by now extremely vast, the one on the effect of trade openness on individual income volatility is much more scant.[1]

The present paper, introducing uncertainty in the classical Ricardian continuum of goods model and focusing on the effects of trade integration on individual welfare, is a contribution to this latter line of research. In the standard (deterministic) Ricardian continuum of goods model, higher trade integration increases efficiency in both country and world production and benefits consumers via the consequent price reduction. Thus, whatever it is its initial level, a tariff reduction is always welfare increasing and the optimal level of protection is zero. The present model formalizes the intuition that, instead, if there is uncertainty and jobs are characterized by a positive level of specificity, changing the level of protection entails *both* costs and benefits.

This *trade off* is captured in the model in a very simple way. In each period the realization of a stochastic variable determines the range of domestically produced varieties. The presence of uncertainty concerning country's comparative advantages also implies that in each period there is a positive probability for workers to be displaced. The latter it is higher 1) the lower the difference in the relative sectoral productivities between the foreign and the domestic country; 2) the closer the sector the worker is employed into is to the *borderline* one. In case of displacement, since each job is characterized by a positive degree of specificity, the worker suffers a loss because moving from her sector to another one is costly. Under the assumption that the more (fewer) the sectors, the lower (higher) the cost to find a new job when displaced, I obtain the full characterization of

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[1] Until now, the only paper that has addressed this question from an empirical point of view is Krebs et alt. (2005).

the effect of higher trade integration on worker's expected income. In the model, protecting the economy with an import tariff is costly in that there is no full exploitation of the possible efficiency gains and of the related price reduction. But protection also reduces income loss in case of displacement and thus increases expected welfare of workers[2]. The main result of the paper is that, depending on the economic structural characteristics of the country and under proper limitation of the parameters' space, increasing trade integration may decrease expected utility.

There are two main sources of inspiration for the present work. The first is the trade under uncertainty literature. Since the pioneering contribution by Brainard and Copper (1968), this line of research has derived a series of important (because unconventional) theoretical propositions, most of which are in open contradiction with classical ones. As it is well known, one of the fundamental result of classical trade theory is that under perfect competition and in the absence of external economies, free trade leads, through promoting proper specialization, to efficiency in world production (MacKenzie, 1954; Dornbusch et alt. 1977). But under uncertainty this is not more true. Indeed, under uncertainty the optimal country specialization level is lower than in a deterministic setting and trade theorems (i.e. the factor-price equalization theorem, the Stolper-Samuelson theorem, the Rybczynski theorem, Heckscher-Ohlin theorem, etc..) do not hold in the absence of complete international asset markets[3]. In addition Kemp and Liviatan (1973), Ruffin (1974) and Turnovsky (1974) demonstrated that in a Ricardian two-sector model the (optimal) pattern of trade does not need to follow comparative cost advantages, and thus that the doctrine of comparative advantages does not work properly under uncertainty. But, no attempt has been made to link these results to how the presence of uncertainty modifies the effect of increasing trade integration, and in particular higher specialization, on individual well-being[4]. One of the objectives of this paper is indeed to fill this gap.

The second source of inspiration for this paper is the literature on the optimality

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[2]While in Eaton and Grossman (1985) tariff revenues are used to compensate workers employed in the unlucky sector, in this model, on the contrary, the only effect of tariff protection is the provision of a larger number of active sector, that work as a risk reducing device in case of negative shock affecting *all* sectors.

[3]For excellent surveys of these results see Helpman and Razin (1978) and Hoff (1994).

[4]A partial exception is Rodrik (1997). Indeed, while some authors have emphasized the stabilizing effect (of both prices and quantities) of more integrated and larger product markets, he shows that, when stronger foreign competition increases the elasticity of labor demand functions, any given shock would translate into larger variations in wages and employment and thus in more volatile incomes.

of government intervention in a trade context under uncertainty. Eaton and Grossman (1985) pioneered this literature exploring the use of government-imposed trade tariff protection as a substitute for missing insurance markets. They show that, in the presence of a specific factor and with incomplete markets for contingent claims, free trade is not optimal and that government can improve social welfare by using commercial policy. Government intervention can also be the instrument to achieve the optimal level of specialization. Brainard (1991) presents a two-sector Ricardian model with specific workers in which taxes and transfers can be used to induce risk averse workers to specialize optimally from a social point of view, a result that would not be achievable in the absence of government intervention. Bowles and Pagano (2006) emphasizes how the degree of worker's specificity is an important variable to determine her preferences in choosing between higher trade integration or stronger government intervention, e.g. through the provision of a tax-based insurance mechanism, in the economy. All these models thus show that an instance in which government intervention may be welfare increasing is when (at least) one of the production factors is characterized by a positive degree of specificity. A peculiar feature of the present model is that, differently from previous ones, the degree of worker's specificity is *endogenously* determined in the model and depends on the specialization level of the country. Furthermore the change in the number of active sectors in the economy also modifies the cost workers' suffer in case of displacement: *ceteris paribus*, the smaller number of domestic active sectors the higher the cost. Finally, worker's level of risk exposure also depends on its occupational sectoral location[5].

The Ricardian continuum of goods model has been widely used in the literature but mostly in a deterministic setting[6]. To the best of my knowledge, this is the first stochastic version of it that it is used to analyze the effects of higher trade integration on individual welfare. While the result that under uncertainty higher trade integration may lead to lower welfare is not novel to the literature (Newbery and Stiglitz, 1984), the original contribution of this paper is to provide a simple model in which the benefits and cost of increasing trade integration are modeled together and to derive the conditions

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[5]Empirical results for the Mexican case show that trade policy changes have a significant short run effect on income risk for industries with high levels of import penetration. Krebs et alt. (2005) calculate that a 5% tariff reduction the standard deviation of the persistent shocks to income by about 25%.

[6]Eaton and Kortum (2002) provide a multi-country stochastic version of the Ricardian continuum of goods model. The present model differs from that because I model uncertainty in a different way and the focus is on individual welfare rather than on trade flows.

under which an *optimal* positive level of protection exists.

The paper is structured as follows. In Section 2, the building blocks of the model are presented. In Section 3, I describe the effects of increasing trade integration on individual welfare and the two main results of the paper are derived and discussed. Section 4 concludes.

2   The model

In this section I present a variant of the classical Ricardian continuum of goods model modified by the introduction of uncertainty[7].

2.1   Supply

Consider two countries, South and North. Both countries can produce a set of goods indexed by z, modeled as a continuum on an unit interval. Thus we have $z \in [0, 1]$. In South the production of sector z is described by a Cobb-Douglas production function

$$Y_z = a(z)K_z^\alpha L_z^{1-\alpha} \tag{1}$$

where $a(z)$ is the sector specific productivity parameter.

Perfectly competitive firms produce variety z combining capital (K) and labour (L) using the constant return to scale technology (1), so that the producers of variety z choose K_z and L_z to maximize their profits

$$\Pi_z = p_z a(z)K_z^\alpha L_z^{1-\alpha} - w_z L_z - rK_z$$

where $\alpha \in [0, 1]$, w_z is the wage rate paid in sector z, and r is the exogenously given world rate of interest. Firms chose the optimal labour capital ratio is sector z which is given by:

$$\frac{L_z}{K_z} = \frac{1-\alpha}{\alpha}\frac{r}{w_z} \tag{2}$$

For the sake of simplicity, I make the following:

Assumption 1 *Capital flows into the economy as to maintain full employment.*[8]

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[7]More precisely I introduce uncertainty in the Imbs and Warcziarg (2000) version of the Dornbusch et alt (1977) model.

[8]If capital was constant, any reduction in the tariff protection would produce unemployment. As it I will show below (section 2.4), a reduction of t (and thus an increase of z_i) implies an increase of w_z. It

Substituting (2) into the FOC for the maximisation of profits, the South's price for each variety of the continuum is determined and it is given by

$$p_z = \frac{1}{a(z)} \frac{r}{\alpha} \left(\frac{r}{w_z} \frac{1-\alpha}{\alpha} \right)^{\alpha-1} \tag{3}$$

Assume that production technologies are identical but for the sectors specific parameter and that that $a(z) \neq a^*(z)$ for all z, where $a^*(z)$ is North's sector specific productivity parameter. Rank the goods in order to have $A'(z) < 0$, where $A(z) = \frac{a^*(z)}{a(z)}$. This simply means that the goods are ranked from the one in which the South productivity comparative advantage is lower to the one in which it is higher.

Assume now that the comparative advantages, differently from the standard model, are stochastic (or, alternatively, that they not perfectly known by the agents). Since only relative productivities matter, I assume that South productivity is deterministic (perfectly known), while the one in North it is not. I model this uncertainty in the form of a multiplicative parameter $\theta \sim U(0,1)$. Thus I have

$$A(z) = \frac{(1+\theta) a^*(z)}{a(z)} \tag{4}$$

Note that the higher θ, the higher North relative productivity.

Assumption 2 *The stochastic parameter θ does not modifies the rank of the comparative advantages.*

2.2 Demand

In order to minimize the effects of demand driven phenomena, I assume that all the agents are characterised by Leontief preferences over the different varieties. This implies that the demand for each good is the same. Agents derive utility from the consumption of all varieties and the domestic and foreign produced goods of the same variety are perfect substitute.

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follows that in each 'ex-post' active sector the equilibrium wage rate is higher and (from equation (2)) labour demand is lower. This implies that total labour demand L_d defined as

$$L_d = \int_{z_i}^{1} L_z dz$$

decreases, *ceteris paribus*, with trade integration. Note that since the objective of the paper is to derive the conditions under which a tariff reduction is welfare decreasing, Assumption 1 makes the achievement of this objective harder, not simpler. Indeed, if this assumption did not hold, *ceteris paribus*, the *cost* of reducing tariff protection would be larger.

Each period, agents in South choose $\mathbf{X} = [X_z]$ (the vector of consumption) to maximize:

$$min\left(X_0, \ldots, X_1\right)$$

$$s.t. \quad \int_0^{z_i} (1+t)\frac{p_z^*}{1+\theta}X_z dz + \int_{z_i}^1 p_z X_z dz = Y_s$$

where the symbol * refers to the foreign country's variables, p_z is the price of good z in South, X_z is the consumption of good z, t is South import tariff, Y_s is total South nominal expenditure and z_i is the threshold goods that determines the import set. The domestic economy produces all the varieties belonging to the interval $[z_i, 1]$. Note that the uncertainty concerning foreign competitiveness implies that both z_i and Y_s are stochastic. Since their actual values depend on the realization of θ, also P is a stochastic variable. Given Leontief preferences, for all z, $X_z = X$ and the budget constraint becomes $PX = Y_s$, where

$$P = \int_0^{z_i} (1+t)E\left(p_z^*\right)dz + \int_{z_i}^1 p_z dz$$

is the domestic price index and $E(\cdot)$ the expectation operator. By analogy, in the foreign country the budget constraint reads:

$$X^*\left[\int_0^{z_e} E\left(p_z^*\right)dz + \int_{z_e}^1 (1+t^*)p_z dz\right] = Y^*$$

where $[0; z_e]$ represent the range of foreign produced goods.

2.3 The specialization pattern

Differently from the deterministic case, the pattern of international specialization is determined by the realization of a random parameter. Consider the case in which, after the realization of the shock, the ex-post price of a good previously produced in South is now lower in North. Depending on the *value* of the realization of the θ parameter two different outcomes are possible. First, the sector close down and workers are displaced. Second, workers accept a wage reduction sufficient to maintain the sector competitive with respect to the foreign competitor. Indeed, since they are specific, after uncertainty resolves, they would accept a reduction in their wage up to the moving cost s instead of leaving their sector.

The sector will close down if

$$p_z > \frac{1+t}{1+\theta}\Gamma - s$$

The cut-off value of the stochastic parameter is:

$$\theta > \bar{\theta} = \frac{(1+t)\Gamma}{p_z - s} - 1 \tag{5}$$

where s is the worker's moving cost. Thus, the realization of the stochastic variable produces the displacement of the workers (and thus a change in the specialization level) only if it is sufficiently high[9]. Instead, for $\theta < \bar{\theta}$, it would be optimal for workers not to leave the sector and to accept the wage reduction needed to maintain the sector competitive since the moving cost (the specificity cost) is larger than the wage loss. The value of θ for which there is displacement is, as expected, higher i) the higher the North's price; ii) the lower (i.e. the more competitive) is South's wage; iii) the higher South's protection. In addition, the higher South's sectoral productivities the higher the value of $\bar{\theta}$[10]. Conversely, the closer the sector is to the borderline one, i.e. the smaller the cross-country difference in sectoral productivities, the lower the level of the shock for which there is displacement.

Since the focus of the model is on how the changes in the specialization pattern provoked by the presence of uncertainty affects individual welfare, I limit the analysis to case in which the parameter assumes only two values: either $\theta = 0$ or $\theta > \bar{\theta}$[11].

Thus, South imports good z if and only if

$$p_z > \frac{(1+t)}{(1+\hat{\theta})} p_z^*$$

where t is an uniform *ad valorem* tariff and $\hat{\theta}$ is the realization of the stochastic parameter. The equality defines the Souths borderline import good z_i. Assuming $p_z^* = \Gamma$ we have that:

$$p_z = \frac{(1+t)}{1+\hat{\theta}} \Gamma$$

$$a(z) = \frac{1}{(1+t)} \frac{r}{\alpha} \left(\frac{r}{w_z} \frac{1-\alpha}{\alpha} \right)^{\alpha-1} \frac{1+\hat{\theta}}{\Gamma}$$

If $a(z)$ is an invertible function, the borderline good is given by:

$$z_i = a^{-1} \left[\frac{1}{(1+t)} \frac{r}{\alpha} \left(\frac{r}{w_{z_i}} \frac{1-\alpha}{\alpha} \right)^{\alpha-1} \frac{1+\hat{\theta}}{\Gamma} \right] \tag{6}$$

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[9]Were the wages assumed to be rigid downward, there would be displacement for any *theta*.

[10]See the Appendix 1 for the derivation

[11]Note that, even when $0 < \theta < \bar{\theta}$ and there is no displacement, protection may play a positive role reducing the probability of wage cut for marginal workers. Were the workers risk averse, the positive effect of protection, i.e. the reduction of wage variability, would be increasing in the workers' degree of risk aversion.
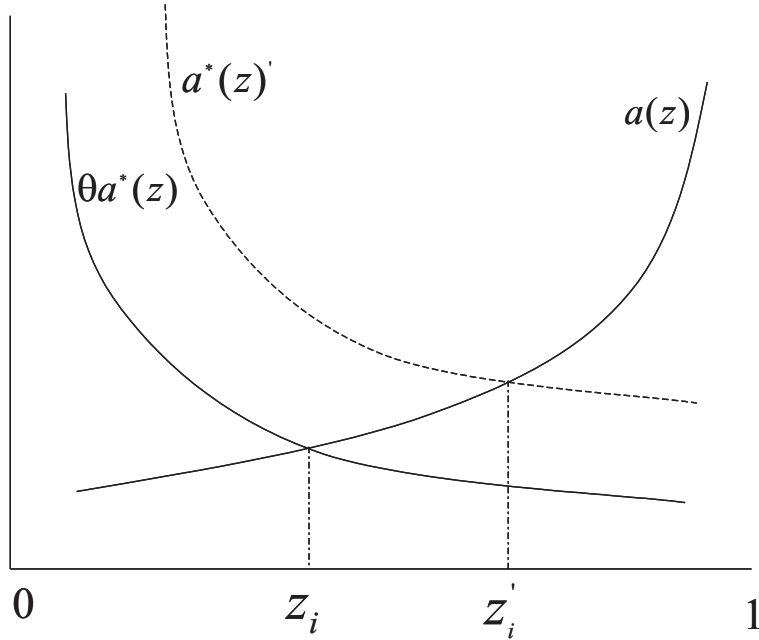
Figure 1: The figure depicts two, among the many, functions $a(z)$ and $a'(z)$ for which $A'(z) = \frac{(1+\theta)a^*(z)}{a(z)} < 0$. The curve $a^*(z)'$ represents the *ex-post* schedule of the foreign country labour productivity when $\hat{\theta} > \bar{\theta}$, i.e. the *whole* schedule of the foreign country productivity has shifted to the right after uncertainty resolved. The new borderline good z_i' is identified by the intersection between the domestic curve and the new foreign one. As implied by (7), in this case the number of domestically produced goods decreases.

This expression determines the threshold good z_i. South imports all the varieties in the range $[0, z_i]$. From (6) it immediately follows that

$$\frac{\partial z_i}{\partial \theta} > 0 \qquad and \qquad \frac{\partial z_i}{\partial t} < 0 \tag{7}$$

with $\theta > \bar{\theta}$. Thus, for given wage rate, the higher θ the smaller the set of goods for which South enjoys a comparative advantage, i.e. the smaller the set of domestically produced goods. If t decreases (i.e. South increases its openness), z_i increases, i.e. the number of imported varieties increases and the number of the domestically produced ones decreases. In a Ricardian model with a continuum of goods, this is equivalent to say that if t decreases the specialization level of South increases. Figure 1 exemplifies the effect of the realization of the stochastic parameter on the specialization pattern.

In a specular way, it is determined the range of exported goods. Since the focus of the paper is on South's trade policy behavior, in the following I will assume that North is already totally opened, i.e. $t^* = 0$. This implies that South exports good z if and only
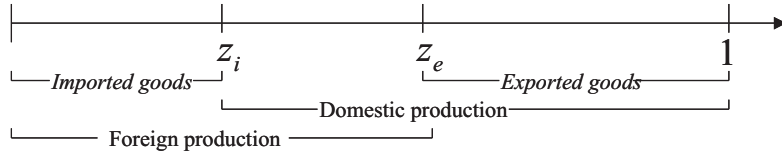
8

Figure 2: The continuum of goods and the international division of labour

if $p_z < \hat{\theta}\Gamma$. The equality defines the Souths borderline import good z_e. Thus,

$$p_z = \frac{\Gamma}{1 + \hat{\theta}}$$

$$a(z) = \frac{r}{\alpha}\left(\frac{r}{w_z}\frac{1-\alpha}{\alpha}\right)^{\alpha-1}\frac{1+\hat{\theta}}{\Gamma}$$

Finally, if $a(z)$ is invertible, I can write:

$$z_e = a^{-1}\left[\frac{r}{\alpha}\left(\frac{r}{w_{z_e}}\frac{1-\alpha}{\alpha}\right)^{\alpha-1}\frac{1+\hat{\theta}}{\Gamma}\right]$$

Thus this expression determines the threshold good z_e. South will export varieties in the range $[z_e, 1]$. It is clear that for $t > 0$ we have that $z_e > z_i$. This implies that there is a range of non-traded goods $[z_i, z_e]$ that both countries produce but that they do not trade (see Figure 2). The impact of a reduction of t is indeed the contraction of this latter set of goods.

2.4 Sectoral equilibrium wage

Similarly to Bowles and Pagano (2006), in the present setting uncertainty takes the form of the occurrence of either a status quo state, in which the individual continues to work in her sector earning a wage w_z, or a bad state in which there is no demand for the good produced in the sector the worker is employed into. In the latter case the worker must move to another sector. There her wage will be $(1-s)w_z$, where s is a measure of the degree to which her skills are specific to the initial livelihood[12]. Thus, expected income in sector z is

$$E(I_{hz}) = \pi w_z + (1-\pi)(1-s)w_z \tag{8}$$

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[12]Indeed, acquiring the (new) skills appropriate in the destination sector may be costly and time consuming, and that may take the form of foregone wages (Dennis and Iscan, 2005). On this see also Krebs et alt (2005).

where I_h is income of individual h, π is the probability of the status quo and s is a measure of specificity.[13]

Workers in South are risk neutral[14] and identical, thus their preferences can be conveniently be represented by a linear utility function. They are indifferent to the sector to be employed into if and only if their expected utility of income is equalized among the different sectors. Given the general equilibrium nature of the model (captured by the balance of payment equilibrium condition), at least one sector in South is always active, independently from the realization of θ. This sector is sector $z = 1$. This implies that the sectoral wage there, i.e. w_1 - is not random. For the sake of simplicity, also assume that its level is exogenously given[15]. Thus the equilibrium allocation condition reads:

$$
\begin{aligned}
E[U\,(I_i)] &= E[U\,(I_j)] \\
w_1/P &= \pi(w_z/P) + (1 - \pi)(1 - s)(w_z/P)
\end{aligned}
\tag{9}
$$

If condition (9) does not hold no worker would accept to work in sector z. Solving for the equilibrium sectoral wages it yields:

$$
w_z = \frac{w_1}{(1 - s) + \pi s}
\tag{10}
$$

Note that for $s = 0$, $w_z = w_1$. This means that, if agents are not specific, wages are equalized across sectors and the model collapses in the standard Ricardian continuum of goods model. In addition, if $\pi_1 = 1$ $w_z = w_1$, i.e. if the probability of displacement is zero, wages are equalized across sectors.

The effect of changes in the parameters on the equilibrium sectoral wage are straightforward and intuitive. The sectoral equilibrium wage is higher: 1) the lower π; 2) the

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[13]A more complicated but general formulation can be:

$$
E(I_{hz}) = \pi w_z + (1 - \pi)\hat{w}_{-z}(1 - s)
$$

where

$$
\hat{w}_{-z} = \int_{z_i}^{1} \phi_z w_z dz
$$

where ϕ_j is the probability to go sector z and w_z is sectoral wage. As it will be shown later, since wages include a compensation for risk to equalize expected utilities of workers in different sectors, the wages are all 'equal' and thus the wage in the destination sector (whatever it is) is equivalent to w_z.

[14]Note that in the case of risk averse workers our argument would be just reinforced.

[15]For instance, one can assume that it is the result of a bargaining process (for simplicity not modeled here) between capitalist and workers.

higher w_1; 3) the higher the specificity of workers[16]. Note that (10) implies

$$E(I_{hz}) = w_1$$

In North there is no uncertainty and thus there is only one common wage for all sectors, w^*.

I now add some more structure to the model in order to analytically evaluate the effects of a reduction in the tariff level on welfare.

Specificity Since worker h is specific to her sector, this implies that when she had to move to a new job because of a bad state, part of her competencies will be useless in the new sector. As shown in equation (8), the higher her specificity (i.e. the moving cost) the larger the wage reduction. I assume that the specificity (s) is a *decreasing* function of the number of sectors. There are two possible justifications for this. The first is based on the idea that the smaller the range of domestically active sectors the higher the average *technological distance* worker h has to travel to find a new job. The second applies when production specialization is correlated with regional specialization. In this case, when a sector disappears, workers have to physically move towards another location. Thus, the fewer the sectors, the larger the 'travel' and the moving costs. To model the idea that the income loss is decreasing in the number of domestic active sectors I assume that:

$$s = f(z_i)$$

with

$$\frac{\partial s}{\partial z_i} > 0 \tag{11}$$

The simplest functional form that satisfies (11) is

$$s = z_i \tag{12}$$

Equation (12) states that if z_i increases, the wage loss in case of bad state increases.

Probability of displacement The equilibrium sectoral wage depends also on the probability of displacement $(1 - \pi)$. Sectors are differently exposed to risk. In particular, the closer the sector the worker is employed into is to the borderline sector z_i, the higher

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[16]Differentiating (10), it yields:
$$\frac{\partial w_z}{\partial s} = \frac{w_1(1 - \pi)}{[1 - s(1 - \pi)]^2} > 0$$

the higher the probability that it can be wiped-out by foreign competition. I incorporate this feature explicitly in the model assuming that the probability of displacement is sectoral dependent. To make things simple, I assume that

$$\frac{\partial \pi}{\partial z} > 0$$

This probability clearly depends on (i) the realization of the random event, i.e. if $\theta > \bar{\theta}$; (ii) the shape of the A curve; (iii) the sector the worker is employed into. Choosing a specific functional form:

$$\pi_z = f(z) = \gamma z \tag{13}$$

where $0 < \gamma < 1$ is a positive constant. Equation (13) states that the probability of being displaced decreases as we move toward the sectors in which South has stronger comparative advantages. Note that the chosen functional form allows for across country comparisons. *Ceteris paribus*, if a country is characterized by a higher γ, this means that for given shock the probability that workers have to leave any sector is lower. Indeed γ implicitly describes different patterns of relative comparative advantages (i.e. different slopes (shapes) of the $A(z)$ curve). This obviously implies that the equilibrium sectoral wage rate is a decreasing function of γ.

Substituting (12) and (13) into (10), the equilibrium sectoral wage can be rewritten as:

$$w_z = \frac{w_1}{(1 - z_i) + \gamma z z_i} \tag{14}$$

Thus equilibrium condition entails a compensation for risk. The lower z, the lower the (margin) of comparative advantage of South. This increases the probability that, for given positive realization of θ, the relative North productivity is higher, making good z an import rather than an export for South.

Here they are evident the conflicting effects of a reduction of t, i.e. of an increase of specialization. On the one hand a reduction of the protection increases the average wage w_z due to the increase of s. On the other, since π_z depends positively on z_i, a reduction of t reduces the equilibrium wage (because the probability of remaining in the 'survived' sectors is *now* higher) making the economy more competitive.

2.5   Trade equilibrium

Under the assumption of Leontief preferences the per period trade balance condition reads

$$\int_{z_e}^1 p_z X^* dz = \int_0^{z_i} (1 + t) \frac{\Gamma}{1 + \theta} X dz$$

Under uncertainty it is clear that the equilibrium can be only in expected terms.

2.6 Price index

Recall that the price index has been defined as:

$$P = (1+t)\frac{\Gamma}{1+\theta}z_i + \int_{z_i}^{1} p_z dz \qquad (15)$$

The first term on the right-hand side represents the component of the price index that depends on the price and number of imported varieties. The second one refers to the domestic component of the index and is given by the integral of the prices paid for domestically produced goods.

Substituting equation (3) and (6) into (15) the expected domestic price index can be rewritten as[17]:

$$P = \Phi \left[w_{z_i}^{1-\alpha} + \int_{z_i}^{1} \frac{w_z^{1-\alpha}}{a(z)} dz \right]$$

where Φ is a positive constant.

3 Optimal protection

In the standard Ricardian continuum of goods model the optimal level of the tariff is zero. Since the model is based on comparative advantages and there is perfect competition, free trade is always optimal. Indeed free trade, allowing the efficient allocation of world production, maximizes both country and world welfare. Any tariff protection would distort the optimal international division of labour reducing income.

In the presence of uncertainty and specificity of workers this is not true anymore. Differently from what happens in the deterministic case, under uncertainty reducing trade protection has both a positive and negative effect on workers' utility. Indeed there is a trade-off between higher average expected wage and higher cost in case of displacement - where the latter is the consequence of an increase in the specialization level. When the 'displacement cost' effect is stronger than the efficiency gains effect, free trade is not optimal. Using the simple model I have introduced in the previous section, in the following I derive the conditions under which the optimal level of protection is positive.

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[17]See Appendix 1 for the derivation.

The positive effect of trade integration   Tariff reduction has two opposite effects on the price index. The first is a (direct) positive one: the set of imported varieties $[0; z_i]$ becomes less expensive if t decreases. But at the same time, as t decreases z_i increases, enlarging the number of varieties that are imported. In order to determine the net effect of a reduction of protection on the domestic price index, I need to derive a closed analytical expression for P. For this reason I have to assume an explicit functional form for the pattern of sectoral labour productivities in South. For simplicity let $a(z) = z$. Thus the price index is South reads[18]:

$$P = \Phi \left[w_{z_i}^{1-\alpha} + \int_{z_i}^1 \frac{w_z^{1-\alpha}}{z} dz \right] \tag{16}$$

To begin with, note that the second term in the right-hand side of equation (16) is always decreasing with a tariff reduction, because (from (7)) a lower t implies a higher z_i.

Then I consider the first term. This part of the equation captures the cost associated with importing foreign produced varieties on which South impose a tariff. Recalling the expression for sectoral wage (equation 14) and differentiating it with respect to z_i, I obtain:

$$\frac{\partial w_{z_i}^{1-\alpha}}{\partial z_i} = w_1^{1-\alpha} \frac{(1-\alpha)\left[1 - 2\gamma z_i\right]}{\left[(1 - z_i) + \gamma z_i^2\right]^{2-\alpha}} \tag{17}$$

Since the sign of (17) depends only on the sign of numerator we have:

$$\frac{\partial w_{z_i}^{1-\alpha}}{\partial z_i} > 0 \qquad iff \qquad (1-\alpha)\left[1 - 2\gamma z_i\right] > 0 \tag{18}$$

The condition for which an increase in specialization reduces the first term of (16) is given by:[19]

$$z_i^* > \frac{1}{2\gamma} \tag{19}$$

where z_i^* gives the lower bound value of the borderline goods sufficient for a tariff reduction to reduce the price index. Under the restriction $\gamma < 0.5$, the first term of (16) is always increasing with z_i.[20] This result means that if the probability of displacement is

---

[18]Assuming $a(z) = z^\beta$ equation (15) would become

$$P = \left[(1+t)\frac{\Gamma}{1+\theta} \right]^{\frac{\beta-1}{\beta}} w_{z_i}^\phi + \int_{z_i}^1 \left(\frac{w_{z_i}^\phi}{z} \right)^\beta dz$$

[19]See Appendix 1.

[20]Note that (19) imposes a lower bound to the value of γ. The necessary condition for $z_i^* < 1$ is $\gamma < 0.5$.

very high, it is possible that the effect of trade integration is an increase in the domestic price level.

Interestingly, it also emerges a non-linear relationship between the tariff level and the price index. First, note that $\frac{\partial z_i^*}{\partial \gamma} < 0$ and thus the lower the γ the higher z_i^*. In other words, the lower γ the smaller the range of specialization starting points from which a reduction in the tariff level reduces the domestic price index. If, on the contrary, γ is very high it is 'easier' that the country benefits from liberalization. In addition note that, *ceteris paribus* a higher γ implies (i) a larger reduction of the price for any given tariff reduction; (ii) a lower the price index for each given level of specialization z_i.

In the standard Ricardian continuum of goods model, the specialization-induced reduction of the price index is the channel through which higher trade integration yields higher real wages and aggregate income (see for a similar result Andersen and Skanksen, 2005). In this model, the same positive effect is present but it is parametrically restricted. Thus, if equation (19), that states the sufficient condition for the price index to decrease after tariff reduction, is satisfied I always have that:

$$\frac{\partial P}{\partial t} > 0 \qquad (20)$$

Consider now the expected income change following a tariff reduction. Given the equilibrium condition I have

$$\frac{\partial E(I_{hz}/P)}{\partial t} = w_1 \left(\frac{\partial P}{\partial t} \right)^{-1} < 0$$

This implies that as the tariff decreases expected workers' income increases. Thus considering just expected income the optimal level of tariff would be $t^* = 0$.

3.1 Trade integration and expected utility

As we have seen the positive effect of reducing trade protection is given by the reduction of the price index. This increases workers' real wage. I have also shown that this positive effect is higher the higher γ, i.e. the steeper the function $A(z)$. This implies that the more different are the two countries, the higher the benefit of increasing trade integration. As discussed in section 2.4 the negative effect of reducing protection is related to the fact that the number of active domestic sector decreases, increasing the wage loss in case of displacement.

Now consider what is the effect of increasing trade integration on the expected utility of a worker employed in sector z combining these two effects. Substitute (12) and (13)

into equation (8), to obtain:

$$E(U_z) = E\left[\pi(w_z/P) + (1-\pi)(1-z_i)(w_z/P)\right] \tag{21}$$

3.1.1 Slow adjustment

Lets begin considering the case in which the sectoral wage adjust with a lag to the (ex-post) new equilibrium. In this case w_z is assumed not to change with t. Differentiating (21) with respect to t, it yields:

$$\frac{\partial E(U_z)}{\partial t} = \left[\pi w_z \left(\frac{\partial P}{\partial t}\right)^{-1} - \frac{\partial z_i}{\partial t}(1-\pi)\frac{w_z}{P} + (1-\pi)(1-z_i)w_z\left(\frac{\partial P}{\partial t}\right)^{-1}\right] \tag{22}$$

Note that the two last terms in the right hand side of the equation represent the component of cost related to the positive probability of displacement

The following propositions contain the two main results of the paper.

Proposition 1 *Define $\phi \equiv \left(\frac{\partial P}{\partial t}\right)^{-1}$. There exist a negative constant $\bar{\delta}$ such as, $\forall \delta < \bar{\delta}$, if $\phi < \delta$ the expected variation of income following a reduction of the tariff level is positive.*

Proof. See Appendix 2 ∎

Proposition 1 states that the expected variation of income following a reduction in tariff is positive only if the reduction in the price index is sufficiently strong. Otherwise the negative effect of the increase in risk (due to the reduction of the number of active sectors) and the cost related to displacement make increasing trade integration welfare reducting.

The second result is contained in the following:

Proposition 2 *Define $\tau \equiv \frac{\partial z_i}{\partial t}$. If $\phi < \left(\frac{1-\pi}{P}\right)\tau$, the optimal tariff is positive.*

Proof. See Appendix 2 ∎

Proposition 2 states that under uncertainty, in opposition to the deterministic case, the optimal level of protection t^* it is not always zero but it can be positive. The condition under which this is true is that the negative effect of the change in the number of domestically produced varieties (multiplied by a constant) is larger than the positive price effect. How the parameters affect the level of t^* it is described in the following:

Corollary 1 *When the optimal level of protection is positive and has the form:*

$$t^* = t^*(\phi, \tau, z)$$

with

$$\frac{\partial t^*}{\partial \phi} < 0 \qquad \frac{\partial t^*}{\partial \tau} > 0 \qquad \frac{\partial t^*}{\partial z} < 0 \tag{23}$$

Proof. See Appendix 2 ∎

The interpretation of the partial derivatives is easy and intuitive. The first states that the stronger the positive effect of a reduction of the tariff on the price index the lower the optimal level of tariff. The second means that, *ceteris paribus*, if the specialization pattern is very sensitive to changes in the tariff rate (i.e. foreign competition is very high), the optimal tariff is higher. The third states that the farther from the borderline sector is the one the worker is employed into, the lower the optimal tariff rate[21].

The results derived so far are based on the assumption that there is a lag between tariff reduction and the adjustment in the equilibrium sectoral wage. I now remove this restrictive (although not totally implausible) assumption.

3.1.2 Instantaneous adjustment

In the previous section we have assumed that agents do not internalize that a reduction of the tariff level would change also the equilibrium sectoral wage.

The changes in the wage rate are determined by two opposite forces. On the one hand, there is the effect of the reduction of the number of active sectors. A lower tariff level is associated with a smaller set of active sectors but the associated π (i.e. the probability to maintain the job in case of shock) would higher on average. All the survived sectors are indeed safer because their relative productivities are higher than the foreign country's ones. Thus, this would reduce the level of the sectoral wage. On the other hand, increasing specialization increase the sectoral wage because of the reduction in s. Indeed, for compensating the higher loss the workers would suffer in case

---

[21]Note the difference between these results and the one presented in Fernandez and Rodrik (1991). In their paper, it is shown the existence of a *status quo bias*: under uncertainty concerning the distribution of gains and losses due to trade liberalization, rational forward looking agents may prefer not to open the economy. It is indeed possible that, even if the decision to opening to free trade would be (ex-post) beneficial to the majority, it may not be undertaken. The present model, on the contrary, does not just compare free trade *vs* autarky. Instead it considers the determinants of the optimal level of protection and the conditions under which it is positive.

of displacement sectoral wages increase. In fact, due to risk compensation, the fewer sectors command a higher compensation for specificity.

The net results of these two effects depends on the model's parameters. While the case of an ex-post lower wage would just reinforce the previous results, this is not so obvious in the case the wage increases after trade integration. This latter case is considered in the following:

Proposition 3 *A wage increasing process of trade integration reduces expected income if the effect of the reduction in the number of sectors is larger than the positive effect of price reduction. In addition, the higher the specialization level of the country, the more likely is that a further reduction of protection would reduce expected income.*

Proof. See Appendix 2 ∎

Proposition 3 states that, even in the case in which the equilibrium sectoral wage increases following trade integration, if the effect of the reduction in the number of sectors is stronger than the positive effect of price reduction expected income can decrease because of lower trade protection.

4   Conclusions

This paper presented a simple model showing that, in a stochastic Ricardian model with a continuum of goods in which workers are partially specific, a positive level of protection may be optimal. While in the standard deterministic model reducing trade protection is always welfare increasing (and thus optimal protection is zero), here I have derived the conditions under which, if comparative advantages are stochastic, this is not true anymore. The reason for this unconventional result is that under uncertainty there is a *trade-off* between the benefit and the cost of higher trade integration. In the present model, as in the standard one, the benefit comes in the form of a lower domestic price index that increases real wage. Instead, the cost of lower tariff protection is the increase of the average distance (and thus the wage loss) workers suffer when displaced. While the former effect increases expected real income, the latter increases the loss she suffers in case of displacement. The main result of the paper is the characterization of the optimal level of protection. This is the level of the tariff for which, given the status quo, a further increase of trade integration would produce benefits (i.e the price reduction) that are smaller than its costs. It has been shown that the optimal level of protection

depends on the structural parameters of the economy. *Ceteris paribus*, the smaller the price reduction induced by increasing trade integration, the higher the optimal tariff. The optimal tariff is also increasing in the *overall* degree of foreign competitiveness: this implies that the more technologically similar are the countries, the higher is t^*. Finally, t^* depends on the worker's sectoral location; the lower the sectoral comparative advantage the higher the optimal level of protection.

This model provides a new application of the general result, coming from the trade under uncertainty literature, that increasing specialization entails both benefits and costs. While in the deterministic setting only the firsts are present, under uncertainty the latter may be so relevant that increasing trade integration beyond a certain level may become welfare reducing. The assumption of risk-neutral workers implies that the cost of higher specialization does not depend on the degree of risk aversion but it is related to the risk of displacement. Thus, the cost *does* depend on the structural characteristic of the economy (i.e. the degree of specificity) and not on the preferences of the worker. As it is immediate to understand, were the workers risk averse, the models' results would just be reinforced. In addition, note that the models' results are quite robust because tariff revenues are not redistributed and thus are a pure waste.

Two are the main predictions of the model. First, if the process of trade integration is characterized by an increasing international division of labour, it will encounter increasing opposition due to the fact that its costs increase with production specialization. Second, since, as the process of globalization proceeds, the cost of increasing trade integration (measured by s) is likely to rise more quickly in developing country rather than in developed ones, we should expect higher opposition to it there.

This model shows that cases of opposition to the ongoing processes of higher trade integration may be easily justified once not only the benefits but also their costs are acknowledged. Recent estimation have shown that the positive effect of trade liberalization may be very small (Rodrik, 2006): this evidence supports the view that in many cases, in the absence of any compensating mechanism, it is well possible that (at least in the short run) costs may be bigger than benefits. It is thus evident that increasing trade integration cannot be 'dogmatically' assumed to be always optimal.

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Appendix 1

Derivation of $\bar{\theta}$ Since workers are specific, in case of displacement, they would incur in the (moving) cost s. If, after the shock, the wage reduction needed for the sector to remain competitive (and thus to continue producing good z) is smaller than the moving cost, they will accept a lower wage and will not leave the sector. On the contrary, workers will leave the sector if the difference in the prices (i.e. the wage reduction) is higher than the moving cost. Thus the 'worker's displacement' conditions reads:

$$p_z - (1+t)\frac{\Gamma}{1+\theta} > s$$

$$\theta > \frac{(1+t)\Gamma}{p(z) - s} - 1$$

Noting that the previous condition can be re-written as

$$\theta > \frac{(1+t)\Gamma}{\left[\frac{1}{a(z)}\frac{r}{\alpha}\left(\frac{r}{w_z}\frac{1-\alpha}{\alpha}\right)^{\alpha-1} - s\right]} - 1$$

To show the effect of larger productivity differences on the cut-off value of the stochastic parameter, I differentiate with respect to $a(z)$:

$$\frac{\partial\theta}{\partial a(z)} = (1+t)\frac{\left[\frac{1}{a^2(z)}\frac{r}{\alpha}\left(\frac{r}{w_z}\frac{1-\alpha}{\alpha}\right)^{\alpha-1}\right]}{\left[\frac{1}{a(z)}\frac{r}{\alpha}\left(\frac{r}{w_z}\frac{1-\alpha}{\alpha}\right)^{\alpha-1} - s\right]^2} > 0 \tag{24}$$

Derivation of equation (16) Substituting equation (3) and (6) into (15), the expected domestic price index can be rewritten as:

$$
\begin{aligned}
P &= (1+t)\frac{\Gamma}{1+\theta}z_i + \int_{z_i}^1 p_z dz \\
&= (1+t)\frac{\Gamma}{1+\theta}z_i + \int_{z_i}^1 \frac{1}{a(z)}\frac{r}{\alpha}\left(\frac{r}{w_z}\frac{1-\alpha}{\alpha}\right)^{\alpha-1} dz \\
&= (1+t)\frac{\Gamma}{1+\theta}\left[\frac{1}{(1+t)\alpha}r\left(\frac{r}{w_{z_i}}\frac{1-\alpha}{\alpha}\right)^{\alpha-1}\frac{1+\theta}{\Gamma}\right] + \frac{r}{\alpha}\left(r\frac{1-\alpha}{\alpha}\right)^{\alpha-1}\int_{z_i}^1\frac{w_z^{1-\alpha}}{a(z)}dz \\
&= \left[\frac{r}{\alpha}\left(r\frac{1-\alpha}{\alpha}\right)^{\alpha-1}\right]\left(w_{z_i}^{1-\alpha} + \int_{z_i}^1\frac{w_z^{1-\alpha}}{a(z)}dz\right) \\
P &= \Phi\left[w_{z_i}^{1-\alpha} + \int_{z_i}^1\frac{w_z^{1-\alpha}}{a(z)}dz\right]
\end{aligned}
$$

Derivation of condition (19)   Evaluating (14) at z_i we have

$$w_{z_i} = \frac{w_1}{(1 - z_i) + \gamma z_i^2}$$

Differentiating with respect to z_i it yields

$$\frac{\partial w_{z_i}^{1-\alpha}}{\partial z_i} = w_1^{1-\alpha} \left[(-1 + 2\gamma z_i)(\alpha - 1) \left[(1 - z_i) + \gamma z_i^2 \right]^{\alpha - 2} \right]$$

Thus the sign depends only on $(1 - 2\gamma z_i)$.

Appendix 2

Slow adjustment

Proof of Proposition 1

$$
\begin{aligned}
\frac{\partial E(U_z)}{\partial t} &= \pi w_z \left(\frac{\partial P}{\partial t} \right)^{-1} - \frac{\partial z_i}{\partial t} (1 - \pi) \frac{w_z}{P} + (1 - \pi)(1 - z_i) w_z \left(\frac{\partial P}{\partial t} \right)^{-1} > 0 \\
&= \left(\frac{\partial P}{\partial t} \right)^{-1} w_z \left[z + (1 - \pi)(1 - z_i) \right] - \frac{\partial z_i}{\partial t} (1 - \pi) \frac{w_z}{P} > 0 \\
&= \left(\frac{\partial P}{\partial t} \right)^{-1} \left[\pi + (1 - \pi)(1 - z_i) \right] - \frac{\partial z_i}{\partial t} (1 - \pi) \frac{1}{P} > 0 \\
&= \left(\frac{\partial P}{\partial t} \right)^{-1} \left[\pi + 1 - z_i - \pi + \pi z_i \right] > \frac{\partial z_i}{\partial t} \frac{(1 - z)}{P}
\end{aligned}
$$

Thus an increase in the tariff increase expected income if

$$\left(\frac{\partial P}{\partial t} \right)^{-1} > \frac{\partial z_i}{\partial t} \frac{(1 - \pi)}{P} \frac{1}{1 - z_i + \pi z_i}$$

Conversely, a reduction of the tariff increases expected utility of agent only if

$$\left(\frac{\partial P}{\partial t} \right)^{-1} < \delta$$

where

$$\delta = \frac{\partial z_i}{\partial t} \frac{(1 - \pi)}{P} \frac{1}{1 - z_i + \pi z_i}$$

i.e. the reduction in the price index is sufficiently large[22].

---

[22]Remember that both $\left(\frac{\partial P}{\partial t} \right)^{-1}$ and $\frac{\partial z_i}{\partial t}$ are negative quantities.

Proof of Proposition 2   Rearranging (22), I obtain

$$\left(\frac{\partial P}{\partial t}\right)^{-1} - \frac{\partial z_i}{\partial t}\frac{(1-\pi)}{P}\frac{1}{1-z_i+\gamma zz_i} = 0$$

Defining as before $\left(\frac{\partial P}{\partial t}\right)^{-1} = \phi$ and $\frac{\partial z_i}{\partial t} = \tau$, it can be written:

$$\phi - \tau\frac{(1-\pi)}{P}\frac{1}{1-z_i+\gamma zz_i} = 0$$
$$P\phi(1-z_i+\gamma zz_i) - \tau(1-\pi) = 0$$
$$\phi P - z_i(\phi P - \gamma z\phi P) - \tau(1-\pi) = 0$$

Then

$$\phi P - z_i\phi P(1-\gamma z) - \tau(1-\pi) = 0$$

and finally

$$z_i^* = \frac{\phi P - \tau(1-\pi)}{\phi P(1-\gamma z)} \tag{25}$$

From equation (25) it is clear that under uncertainty the optimal level of protection is higher than under no-uncertainty. Indeed, when $\pi = 1$, i.e. there is no risk of displacement, the optimal level of specialization is higher. Since we know that under no-uncertainty the optimal level of protection is zero and that the borderline good is an inverse function of the tariff (equation (7)), we can conclude that under uncertainty the optimal level of protection is positive. Note that the smaller π the lower z_i^*, i.e. the higher the risk of displacement the lower the optimal level of specialization, and thus the higher the optimal level of protection.

Proof of Corollary 1   Trivial algebra shows that differentiating equation (25) it yields

$$\frac{\partial z_i^*}{\partial \phi} > 0 \qquad \frac{\partial z_i^*}{\partial \tau} < 0 \qquad \frac{\partial z_i^*}{\partial z} > 0$$

Since $\frac{\partial z^*}{\partial t^*} < 0$, this is sufficient to prove Corollary 1.

Instantaneuos adjustment

Differentiating equation (21), I obtain

$$\frac{\partial E(U_z)}{\partial t} = \left\{ \underbrace{\frac{\partial w_z}{\partial t}\frac{\pi}{P}}_{<0} + \underbrace{\pi w_z\left(\frac{\partial P}{\partial t}\right)^{-1}}_{<0} + (1-\pi)\left[\underbrace{-\frac{\partial z_i}{\partial t}\frac{w_z}{P}}_{>0} + (1-z_i)\underbrace{\left(\frac{\partial w_z}{\partial t}\frac{1}{P} + w_z\left(\frac{\partial P}{\partial t}\right)^{-1}\right)}_{<0}\right]\right\}$$

24

This implies that a reduction of the tariff reduces expected income (i.e. $\frac{\partial E(U_z)}{\partial t} < 0$) only if it reduces too much the number of active sectors (i.e. if $\frac{\partial z_i}{\partial t}$ is very big). Note that the higher the specialization level, i.e. z_i, the larger should be the price reduction for insuring that a reduction in protection would increase expected income. At the same time, the higher $\pi = z$, the more easy it is that a reduction in the tariff increases expected income.

Appendix 3

Using an alternative formulation for equation (8)

$$E(I) = \pi w_z + (1 - \pi)(1 - s)\hat{w} \tag{26}$$

where

$$\hat{w} = \frac{1}{1 - z_i} \int_{z_i}^{1} w_z dz$$

Since in this case it is not possible to derive an explicit expression for the equilibrium wage, it is necessary to consider the implicit function:

$$
\begin{aligned}
\Phi &= \pi w_z - w_1 + (1 - \pi)(1 - s)\frac{1}{1 - z_i} \int_{z_i}^{1} w_z dz \\
&= \pi w_z - w_1 + (1 - \pi)(1 - z_i)\frac{1}{1 - z_i} \int_{z_i}^{1} w_z dz \\
&= \pi w_z - w_1 + (1 - \pi) \int_{z_i}^{1} w_z dz
\end{aligned}
$$

Evaluating how the equilibrium wages changes with the level of specialization it yields:

$$\frac{\partial w_z}{\partial z_i} = -\frac{\frac{\partial \Phi}{\partial z_i}}{\frac{\partial \Phi}{\partial w_z}} = -\frac{< 0}{> 0} > 0$$

Thus, the behavior of the equilibrium wage when it is used equation (8) instead of equation (26) is the same.

Behaviour of the average wage

$$
\begin{aligned}
\hat{w} &= \frac{1}{1 - z_i} \int_{z_i}^{1} w_z dz \\
&= \frac{1}{1 - z_i} \int_{z_i}^{1} \frac{w_1}{(1 - z_i) + z_i \gamma z} dz \\
&= \frac{w_1}{(1 - z_i)\gamma z_i} log \frac{1 + (\gamma + 1)z_i}{1 + (\gamma z_i - 1)z_i}
\end{aligned}
$$

Note that

$$\frac{\partial \hat{w}}{\partial z_i} > 0$$
$$\frac{\partial \hat{w}}{\partial \gamma} < 0$$

Thus the average wage increases with specialization and with the probability of displacement.