Optimal Dimension of Transition Probability Matrices for Markov Chain Bootstrapping

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Abstract

While the large portion of the literature on Markov chain (possibly of order higher than one) bootstrap methods has focused on the correct estimation of the transition probabilities, little or no attention has been devoted to the problem of estimating the dimension of the transition probability matrix. Indeed, it is usual to assume that the Markov chain has a one-step memory property and that the state space could not to be clustered, and coincides with the distinct observed values. In this paper we question the opportunity of such a standard approach. In particular we advance a method to jointly estimate the order of the Markov chain and identify a suitable clustering of the states. Indeed in several real life applications the "memory" of many processes extends well over the last observation; in those cases a correct representation of past trajectories requires a significantly richer set than the state space. On the contrary it can sometimes happen that some distinct values do not correspond to really "different states" of a process; this is a common conclusion whenever, for example, a process assuming two distinct values in t is not affected in its distribution in t+1. Such a situation would suggest to reduce the dimension of the transition probability matrix. Our methods are based on solving two optimization problems. More specifically we consider two competing objectives that a researcher will in general pursue when dealing with bootstrapping: preserving the similarity between the observed and the bootstrap series and reducing the probabilities of getting a perfect replication of the original sample. A brief axiomatic discussion is developed to define the desirable properties for such optimal criteria. Two numerical examples are presented to illustrate the method.

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