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Benefits and Perils of Integrated Data Systems in Managing Sustainable Fishing Quotas

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Abstract

The paper examines how auditing and punishment can be employed to manage a common property resource, specifically a fishery, through a regulation scheme that involves pre-assigned multi-annual quotas. The decision to adhere to or exceed the assigned fishing quota is modeled as a two-player game. In the absence of auditing, the model yields noncompliance with the quota, known as the tragedy of the commons, which is the only Nash equilibrium of the game. With the introduction of formal enforcement, which offsets the additional profit from noncompliance, and extensive auditing through an integrated data control system, adherence to the quota becomes the only Nash equilibrium of the model. A dynamic version of this game, with logistic biomass growth and quadratic costs of harvesting, confirms that a digitized and integrated fishery control system may enhance sustainability. However, the study also highlights the potential risks associated with fishing quotas and enforcement that are not properly adjusted for the resource and are not updated as frequently as necessary.

Keywords Fishery · Managing quota · IT control systems · Enforcing

1 Introduction

The effective management of renewable resources, such as fisheries, poses a significant challenge for humanity. This challenge is particularly pronounced with common property resources (CPRs), which can be used by multiple agents and are therefore characterized by externalities in their use. CPRs require appropriate regulatory measures and formal enforcement to ensure sustainability over time and to prevent overexploitation, see Ostrom

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(1990). In the case of fisheries, the exploitation of resources involves complex interactions between biological and social systems. The involvement of diverse stakeholders with differing objectives on highly exploited resources further complicates the issues. According to OECD (2022), fish consumption is expected to increase at a faster rate than meat over the next decade. An effective management approach necessitates precise information on the status of the resources and the alignment of theoretical biological models of resource growth with accurate data on biomass status. To address the challenges of overfishing, which are often exacerbated by overcapacity, illegal fishing, and inadequate management, the European Union (hereafter, EU) has implemented comprehensive management and monitoring activities targeting individual fish species. The European Common Fisheries Policy (hereafter, CFP) encompasses regulations aimed at conserving marine resources and managing European fisheries both within and outside EU waters.

In particular, *"Fisheries management under the CFP is based on the need to ensure environmentally sustainable exploitation of marine biological resources and the long-term viability of the sector. To achieve this objective, the EU has adopted legislation on access to EU waters, the allocation and use of resources, total allowable catches, fishing effort limitation and technical measures.*"[1]

The activities carried out under the CFP involve using mathematical models to estimate the growth of fish stocks, assessing the current stocks of individual fish species, and setting quotas for the Total Allowable Catch (hereafter, TAC) per species to ensure the highest sustainable annual harvest, known as the Maximum Sustainable Yield (hereafter, MSY) for all European fisheries, see Kanik and Kucuksenel (2016). Fishing quotas are assigned to different countries by considering the interests of both fishers and consumers while also aiming to bring the fish stocks toward MSY.[2]

To support regulatory efforts, national and international Monitoring, Control, and Surveillance (hereafter, MCS) programs have been established to oversee actual fishing activities and penalize overfishing. A fisheries certification program, known as CATCH, has been implemented across the sector to complement these regulatory activities. CATCH is an advanced IT tool designed to streamline assessments and verification of catch certificates for fishery products in the EU market.[3] In essence, all fish products entering the EU market must be accompanied by an Electronic Catch Certificate containing details about the catch, its origin, and the fishing vessel. CATCH facilitates a fully digital and paperless workflow, thereby improving traceability and preventing the import of fishery products obtained from Illegal, Unreported, and Unregulated (hereafter, IUU) fishing into the EU market. It is worth noting that globally, IUU fishing has a value exceeding €23 billion annually and poses significant ecological and reproductive risks to marine species, see Obaidullah (2023).

[1] https://www.europarl.europa.eu/erpl-app-public/factsheets/pdf/en/FTU_3.3.2.pdf.

[2] In 2002, the EU, among several countries, agreed to comply with the Plan of Implementation of the Johannesburg World Summit on Sustainable Development United Nations (2002), committing to maintain or restore fish stocks to levels that can produce the MSY. Elleby et al. (2025) estimate the additional global fish production that could be obtained with MSY management of overfished species, while recognizing the limitations associated with MSY. See also on this point Sustainability in action (2020).

[3] Hereafter, by CATCH we refer to the digital information management system for the Catch Certification Scheme that shall be established by the Commission in accordance with Articles 12b, 12c and 12d of Regulation (EU) 2023/2842 of the European Parliament and of the Council of 22 November 2023 amending Council Regulation (EC) No 1224/2009, and amending Council Regulations (EC) No 1967/2006 and (EC) No 1005/2008 and Regulations (EU) 2016/1139, (EU) 2017/2403 and (EU) 2019/473 of the European Parliament and of the Council as regards fisheries control (https://eur-lex.europa.eu/eli/reg/2023/2842/oj).

According to estimates (Agnew et al. 2009), IUU fishing accounts for approximately 20% of reported catches worldwide, with notable variations across regions and types of catches.

To prevent illegal fishing activities, many governments and supranational institutions are taking steps to adopt mechanisms that discourage illegal actions. In the EU, the CATCH system integrates Vessel Monitoring Systems (hereafter, VMSs) data to reduce the likelihood of misconduct. A VMS allows environmental and fisheries regulatory organizations to track and monitor fishing vessels' activities when a vessel's location or conduct indicates IUU fishing. VMSs can be used to monitor vessels in territorial waters or Exclusive Economic Zones (EEZs) (which extend 200 nautical miles from coasts) and applies to various vessel categories, including local fish such as anchovies and scallops, highly migratory species like tuna, and others. VMSs utilize various communication technologies, such as Automatic Identification Systems (AISs) and satellite links, to transmit catch data. Starting from January 2026, CATCH will be mandatory for both operators and EU authorities for the import of fishery products. This requirement comes after the recent revision of the Regulation against IUU Fishing.[4] The EU's regulatory approach is similar to that of other government agencies. For instance, the New Zealand Ministry of Fisheries establishes an annual TAC for each fish stock to maintain the population at a level that allows for MSY. Similar to this, compliance and enforcement measures involve a series of reporting procedures to track fish from fishing vessels to authorized land receivers. Additionally, an at-sea surveillance program is in place, which includes observers on board fishing vessels, see Newell et al. (2005).

Despite the formal commitments made by countries to address the issue of overexploitation of fish stocks, such as the *Pledge for nature to reverse biodiversity loss*,[5] signed by the EU and 88 countries, it is clear that the outcome referred to as Hardin's *tragedy of the commons* still prevails. This result was anticipated by Hardin in Hardin (1968), based on the game theory approach to the problem of shared resources. Worm et al. (2009) estimated that in the EU, 88% of assessed stocks are being fished beyond their MSY, with 30% of these stocks being outside safe biological limits, see for more information Commission of the European Communities (2009). Pauly and Zeller (2016) show that official data underestimate total withdrawals and stress the importance of control systems on the entire fisheries, including small-scale ones. The tragedy of the commons occurs whenever agreements among exploiters to reduce overfishing are not binding and non-compliant behaviors are not adequately monitored and sanctioned. We refer to Jensen et al. (2017) for a review of the literature on compliance in fisheries models.

In this paper, we present a model of resource exploitation, where a regulator has control and enforcement powers. In a simplified scenario, we consider only two exploiters of the resource who are short-sighted (*myopic*) and risk-neutral maximizers. The regulator sets *optimal* fishing quotas and can choose the level of enforcement and penalties for those who exceed the quotas. We first analyze the short-term harvesting game with quotas for different profit functions, in cases with no punishment (the tragedy of the commons or fisherman's dilemma) and with enforcement and penalties. We then establish thresholds for penalties that ensure agent compliance when exploitation and overfishing activities are adequately monitored, which nowadays seems possible with integrated data systems such as CATCH as described earlier.

[4] https://eur-lex.europa.eu/legal-content/EN/TXT/?uri=CELEX:32008R1005.

[5] See https://www.cbd.int/article/leaders-pledge-for-nature.

Next, we extend our analysis to the long-term dynamics of the game where the actions of the agents and the regulator affect the resource, assuming logistic growth. Exploiters can choose to adhere to the quota without risk of penalty or exceed it for short-term gain. We examine scenarios where the resource is not fished, partial MSY exploitation occurs, and where agents behave like Cournot-Nash-style exploiters. We recall that the equilibrium with constant MSY harvesting does not guarantee resource sustainability, as it can result in resource depletion or extinction because it is semi-stable by construction, see, e.g., Kar and Legovic (2014) and Mesnil (2014) for a critical review on MSY. Under Cournot-Nash harvesting, even more resource depletion or extinction can occur compared to harvesting under the MSY target.

We then tackle a dynamic version of the harvesting game in the case of full compliance, where significant penalties are enforced by the regulator. Mathematically, the dynamics is modeled through a one-dimensional *piecewise-smooth* map. This is because there is a biomass threshold below which agents will exploit the stock less than imposed by the quota, due to economic profitability considerations (Cournot-Nash harvesting).[6] We fully characterize the system dynamics under full compliance when the quota set by the regulator is at MSY, and also provide a robustness analysis for quotas below MSY in Appendix A.

Here we retrieve that, in line with the literature, implementing a quota system with a total catch at MSY, even with full compliance, does not prevent the risk of the resource becoming extinct in finite time. As recalled before, this is because the equilibrium with MSY harvesting is semi-stable. Unfortunately, the same risk may persist even if agents exploit the resource less than the target requires and choose a Cournot-Nash catch to maximize their profit.

In the final part of the paper, we focus on a comprehensive case where the regulator establishes quotas and enforces penalties for non-compliance. Agents face the decision of adhering to the quota or exceeding it if it leads to greater profitability. Failure to adhere to the quota may result from low biomass, making it unprofitable to harvest while sticking to the quota because of high costs, or from high biomass where the benefits of exceeding the quota outweigh the penalties. This last occurrence illustrates a significant risk associated with fines for quota overfishing: if the penalties are not appropriately set, we may observe fluctuations in stock levels followed by overfishing due to low harvesting costs, leading to drastic declines in the resource stock.

We provide a detailed analysis of the dynamic model under enforcement and non-compliance by examining a growth map with both a kink point and a discontinuity point. Through map analysis, we can not only analytically determine the local stability properties of the equilibria, but also identify the initial conditions that lead to convergence to different equilibrium states (i.e., the basins of attraction of attractors). Additionally, we detect more complex dynamics, such as periodic or chaotic behavior. The information on the basins of attraction is particularly important in the context presented, as the size of these basins of attraction serves as a measure of the system's resilience to perturbations in the stock resource. Appropriate staircase diagrams are presented to illustrate the system's dynamics from suitable initial conditions. Bifurcation diagrams show how asymptotic (long-run)

[6] A piecewise-smooth map is a map where the phase space is partitioned into at least two open regions with different smooth functions applying in each region. A piecewise-smooth map may be discontinuous across boundaries, or it may be continuous but not differentiable. See, e.g., Glendinning and Jeffrey (2019) and Avrutin et al. (2019).

dynamics and basins of attraction change as the parameter of the natural growth rate of the fish increases. The comparison of these diagrams allows us to provide a global analysis of the impact of the different harvesting schemes in terms of overexploitation and risk of extinction.

The road map of the paper is as follows. Section 2 introduces a general harvesting model under a fishing quota system with auditing and punishment. Section 3 considers the short-run version of the harvesting game and highlights the benefits of an extensive auditing system. Section 4 presents a specific long-run version of the harvesting game, emphasizing the risks of a fishing quota system despite extensive auditing that ensures punishment for non-compliance. Section 5 concludes. Appendix A contains the investigation of the resource growth model under a fishing quota system with full compliance and quota set at a fraction of the MSY. All the proofs are in Appendix B. Appendix C contains the derivation of the harvesting function under a fishing quota system with partial compliance.

2 The Harvesting Game with Quotas

Let us consider a setup where two identical players harvest a renewable resource (fishery) and sell it in a duopoly market. The fishery is regulated by an authority that sets the TAC and allocates this to the two agents through periodic quotas that each harvester can land over a finite time horizon.[7] For the period of allocation of fishing quotas and for whatever level of biomass X, the players have to decide if they want to comply with the quotas or not. Except for that, players are involved in a Cournot duopoly game and are assumed to be Cournot-Nash players in that, given a biomass level, they harvest the quantity consistent with the Cournot-Nash equilibrium.[8]

Time is measured in discrete periods of allocation of fishing quotas, typically one year. Let X_t be the level of biomass at time t and let us denote by $q^{CN}\left(X_t\right)$ the quantity harvested by a Cournot-Nash player. If a player adheres to the quota regulation, her harvesting is $q^{Q}\left(X_t\right)=\min\left\{Q\left(X_t\right),q^{CN}\left(X_t\right)\right\}$, where $Q\left(X_t\right)$ is the quota imposed by the regulator to an individual player.[9] In the following, we denote by $q_i\left(X_t\right)$ the level of harvesting of fisher i, with $i=1,2$, by $q_{-i}\left(X_t\right)$ the level of harvesting of fisher i's opponent, and by $H\left(X_t\right)=q_1\left(X_t\right)+q_2\left(X_t\right)$ the level of total harvesting.

Let us assume that a fishery control system is implemented, where the frequency of controls depends on the level of harvesting.[10] We assume that the number of controls follows a Poisson process N_t with an enforcement intensity (arrival rate) $\lambda\left(H\left(X_t\right)\right)$, which increases

[7] This modeling assumption is inspired by the current EU legislation, see, e.g., https://www.europarl.europa.eu/erpl-app-public/factsheets/pdf/en/FTU_3.3.2.pdf.

[8] Aiming to provide a simplified representation of European fisheries, the players can be considered as two European countries, whose fishers harvest from the same CPR. Hence, we neglect the issue of Individual Transferable Quotas (hereafter, ITQs) assuming that there is already an optimal allocation of quotas within each country. For compliance issues arising in fishery regulated with ITQs, we refer the reader to Copes (1986) and Chavez and Salgado (2005).

[9] We assume the players to be identical so that the quota is the same for the two players. Generalization with asymmetric players could follow a similar approach. Additionally, we consider a general setup where the quota is set based on the biomass level. In Sect. 4, we consider the common real case of a constant quota.

[10] See for instance https://www.europarl.europa.eu/erpl-app-public/factsheets/pdf/en/FTU_3.3.3.pdf

with the total harvesting.[11] Without loss of generality, we set $\bar{\lambda} = 1$, which is the maximum enforcement rate. Controls are not operated when $\lambda\left(H\left(X_t\right)\right) = 0$ for all $H\left(X_t\right)$, while $\lambda\left(H\left(X_t\right)\right) = \bar{\lambda}$, for all $H\left(X_t\right)$, represents the intensity of controls when the system is active. This control system, referred to as *CATCH*, operates similarly to the CATCH system[12] recently imposed by the EU on the fishing vessels to control harvesting activity in real-time using electronic devices connected to the internet.[13] Any less efficient control system than the CATCH technology is represented by $\lambda\left(H\left(X_t\right)\right)$ being an increasing function of $H\left(X_t\right)$ with $\lambda(0) = 0$ and $\lambda\left(H\left(X_t\right)\right) \to \bar{\lambda}$ as $H\left(X_t\right) \to +\infty$.[14]

Based on the assumptions, the probability of a number *k* of controls during a specific unit of time $(t, t+1]$ is given by

$$\mathbb{P}\left(N_{t+1} - N_t = k\right) = \frac{\left[\lambda\left(H\left(X_t\right)\right)\right]^k e^{-\lambda\left(H\left(X_t\right)\right)}}{k!} \tag{2}$$

where $k = 0, 1, 2, \ldots$. Additionally, agents are risk-neutral, as is standard in the economic literature on law enforcement in fisheries, as seen in references such as Suitinen and Andersen (1985) and Nostbakken (2013). Consequently, the expected profit for fisher *i* in the next unit of time $(t, t+1]$ can be expressed as

$$\pi^e_{i,t+1}\left(q_i\left(X_t\right), q_{-i}\left(X_t\right); X_t\right) = \pi_{i,t+1}\left(q_i\left(X_t\right), q_{-i}\left(X_t\right); X_t\right) - F\left(q_i\left(X_t\right); X_t\right)\mathbb{E}\left[N_{t+1} - N_t \middle| H\left(X_t\right)\right] \tag{3}$$

where $\pi_{i,t+1}$ is the (certain) profit function in case of no controls and $F\left(q_i\left(X_t\right); X_t\right)$ is the *penalty* function. We will assume a constant fine, leading to the penalty function taking the form:[15]

$$F\left(q_i\left(X_t\right); X_t\right) = \begin{cases} \mu & \text{if } q_i\left(X_t\right) > q^Q\left(X_t\right) \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

[11] This assumption is based on the principle that increased harvesting leads to reduced biomass, which in turn requires regulation by a public authority. Additionally, higher harvesting activity increases the likelihood of being inspected if regulation is in place.

[12] See https://oceans-and-fisheries.ec.europa.eu/news/eu-fisheries-control-system-gets-major-revamp-2024-01-09_en.

[13] With the CATCH system, it is possible to control any single harvesting activity; therefore, the probability of control is independent of the level of harvesting. Excluding technical issues that make the CATCH system temporarily unavailable, we set $\bar{\lambda} = 1$ in the following.

[14] A possible analytical expression for the enforcement intensity is:

$$\lambda\left(H\left(X\right)\right) = \bar{\lambda}\frac{H\left(X\right)}{1 + H\left(X\right)} \tag{54}$$

with $\bar{\lambda} = 1$.

[15] Focusing on informal enforcement, Nostbakken (2013) adopts a punishment that is a linear and increasing function of the exceeding fishing-quota violation with a constant enforcement intensity. However, when evaluating the benefits of an IT fisheries control system, the enforcement intensity needs to be endogenous. Additionally, for simplicity and without loss of generality, we assume the punishment to be constant. A punishment as in Nostbakken (2013) would lead to similar conclusions. For reviews on regulatory enforcement in fisheries see Nostbakken (2008) and Jensen et al. (2017).

with $\mu > 0$.

Hence, the expected profit of fisher i in (3) can be expressed as follows:

$$\pi^e_{i,t+1}\left(q_i\left(X_t\right), q_{-i}\left(X_t\right); X_t\right) = \pi_{i,t+1}\left(q_i\left(X_t\right), q_{-i}\left(X_t\right); X_t\right) - F\left(q_i\left(X_t\right); X_t\right)\lambda\left(H\left(X_t\right)\right). \quad (5)$$

Then, following the principle of maximizing expected profit, as stated in Eq. (5), the fishers engage in a matrix game at each time t, selecting between the non-compliant strategy $q^{CN}\left(X_t\right)$ and the compliant strategy $q^{Q}\left(X_t\right)$.

To complete the setup, we will assume that for fisher $i \in \{1, 2\}$, the profit π_i is a generic function of the harvesting levels q_i and q_{-i}, satisfying the following properties. We will drop the dependence of X and π on time, as well as the dependence of q_i and q_{-i} on the level of biomass, to emphasize that these properties hold for any time t and any level of biomass X. In the following sections, we will consider general profit functions that satisfy the following standard assumptions.

Assumption 1 *For each agent* $i \in \{1, 2\}$, *for each* $X \geq 0$ *and for all* $q_i + q_{-i} \leq X$, *let*

(A) $\pi_i\left(q_i, q_{-i}; X\right)$ *be continuous in* q_i *and of class* $\mathcal{C}^2$ *in* $\left(q_i, q_{-i}\right)$;
(B) $\frac{\partial^2 \pi_i}{\partial q_i^2}\left(q_i, q_{-i}; X\right)$ *be strictly negative;*
(C) $\frac{\partial^2 \pi_i}{\partial q_i \partial q_{-i}}\left(q_i, q_{-i}; X\right)$ *be strictly negative;*
(D) $\pi_i\left(q_i, q_{-i}; X\right) = \pi_{-i}\left(q_{-i}, q_i; X\right)$ *for all* $\left(q_i, q_{-i}\right)$.

Strictly negative second derivatives in Assumptions 1(B) and 1(C) ensure that marginal profits are decreasing in q_i and q_{-i}. By the implicit function theorem, Assumptions 1(A), 1(B) and 1(C) imply the existence of downward-sloping best-reply functions. These conditions ensure a character of monotonicity to the first derivative in the first order condition and a non-negligible part of the approximation of the first derivative function that is linear for infinitesimal increments on q_i. According to the definition of expected profits in (5), Assumption 1(D) implies $\pi^e_i\left(q_i, q_{-i}; X\right) = \pi^e_{-i}\left(q_{-i}, q_i; X\right)$ for all $\left(q_i, q_{-i}\right)$. This implies that the game under consideration is symmetric, so from now on we can drop the subscript i from the profit function and from the expected profit function. Next, we recall some basic properties of the profit functions for the fishing quota duopoly game.

2.1 Preliminary Results

Consider the model setup described above, where we simplify notation by removing the time dependence of variables. Hence, the biomass level at time t is denoted as X and the profit and expected profit at time t are denoted as π and π^e, respectively. A fisher has two options: either comply with regulations and harvest the quantity $q^{Q}\left(X\right)$, or non-comply with regulations and harvest the quantity $q^{CN}\left(X\right)$. At each time t, fishers' decisions are modeled as a matrix game based on the biomass level X. Before delving into the Nash equilibria of this matrix game, we highlight some important properties concerning agents' profits. The proof of Property 1 is in Appendix B.

Property 1 (Properties of profits). *In the game under consideration, the following relations hold:*

(A) $\pi\left(q^Q(X),q^Q(X);X\right)\geq\pi\left(q^Q(X),q^{CN}(X);X\right)$;
(B) $\pi\left(q^{CN}(X),q^Q(X);X\right)\geq\pi\left(q^{CN}(X),q^{CN}(X);X\right)$;
(C) $\pi\left(q^{CN}(X),q^Q(X);X\right)\geq\pi\left(q^Q(X),q^Q(X);X\right), with$
$\pi\left(q^{CN}(X),q^Q(X);X\right)>\pi\left(q^Q(X),q^Q(X);X\right)$ *for* $q^{CN}(X)\neq q^Q(X)$;
(D) $\pi\left(q^{CN}(X),q^{CN}(X);X\right)\geq\pi\left(q^Q(X),q^{CN}(X);X\right), with$
$\pi\left(q^{CN}(X),q^{CN}(X);X\right)>\pi\left(q^Q(X),q^{CN}(X);X\right)$ *for* $q^{CN}(X)\neq q^Q(X)$;
(E) $\pi\left(q^{CN}(X),q^Q(X);X\right)\geq\pi\left(q^Q(X),q^{CN}(X);X\right)$.

Note that since the quantity

$$\pi^e(q_i(X),q_{-i}(X);X)-\pi(q_i(X),q_{-i}(X);X)=-F(q_i(X);X)\lambda(H(X))$$

does not depend on $q_{-i}(X)$, (A)-(B) in Property 1 hold also for the expected profit function π^e. More specifically, even assuming that $\pi\left(q^{CN}(X),q^Q(X);X\right)=\pi\left(q^{CN}(X),q^{CN}(X);X\right)$, which happens for example when the demand of fish is inelastic and the cost of harvesting does not depend on the opponent's landing, we can have $\pi^e\left(q^{CN}(X),q^Q(X);X\right)>\pi^e\left(q^{CN}(X),q^{CN}(X);X\right)$. This is because the arrival rate $\lambda(H(X))$ is assumed to be increasing in the total harvesting $H(X)$. Instead, (C)-(E) in Property 1 are not guaranteed for the expected profit function π^e.

3 The Short-Run Harvesting Game with Quotas

This section addresses a short-run version of the game in which the biomass is considered constant. Next, in Sect. 4, we will discuss a long-run version of the model as a repeated game, where the biomass evolves according to a specific growth function.
Assuming constant the level of biomass X, the two fishers play the following static game:

		Column Player	
	Strategies	$q^Q(X)$	$q^{CN}(X)$
Row Player	$q^Q(X)$	$(\pi^e(q^Q(X),q^Q(X);X),\pi^e(q^Q(X),q^Q(X);X))$	$(\pi^e(q^Q(X),q^{CN}(X);X),\pi^e(q^{CN}(X),q^Q(X);X))$
	$q^{CN}(X)$	$(\pi^e(q^{CN}(X),q^Q(X);X),\pi^e(q^Q(X),q^{CN}(X);X))$	$(\pi^e(q^{CN}(X),q^{CN}(X);X),\pi^e(q^{CN}(X),q^{CN}(X);X))$

Employing this two-person matrix game, we analyze how the two fishers interact and how their profits are affected by Assumption 1. We first explore a scenario where the control system is not operational. Then, we will examine the same game with a punishment mechanism that follows a Poisson process with a hazard rate dependent on the biomass, as outlined in Sect. 2.

3.1 No Punishment

We begin the analysis with the scenario where there is no punishment, leading to expected profits being the same as actual profits. The outcome without enforcement is agents' non-compliance, or, in other words, the well-known *tragedy of the commons* (see Hardin (1968)). The next proposition (the proof of which is in Appendix B) characterizes this case.

Proposition 1 (The fisher's dilemma/delight). *Consider the short-run harvesting game with quotas when* $\mu = 0$ *(no punishment). For all X,* either $q^Q(X) = q^{CN}(X)$ *or the game has the unique Nash equilibrium* $\left(q^{CN}(X), q^{CN}(X)\right)$. *Moreover, the game is a prisoner's dilemma when* $\pi\left(q^Q(X), q^Q(X); X\right) > \pi\left(q^{CN}(X), q^{CN}(X); X\right)$ *or a prisoner's delight when the reverse inequality holds.*[16]

The investigation conducted so far suggests that managing fishing quotas is similar to dealing with non-compliant strategies in a prisoner's dilemma or a prisoner's delight game under general conditions. Specifically, when there is no control on landings, fishers either play a prisoner's dilemma game or a prisoner's delight game, which in turn affects the level of fish population in the ecosystem when the biomass is harvested. This aspect is analyzed in Sect. 4 by introducing a time dimension to the game and dynamics for the biomass. Here, we explore how auditing and punishment, in the form of a Poisson process with a hazard rate depending on the biomass, can encourage compliance with the regulations in the static game.

3.2 Controls Based on Fishers' Harvesting

If the controls depend on the level of harvesting and the punishment is a positive value denoted as $\mu > 0$, then the expected profit of a fisher who complies with the rules is the same as her actual profit. However, for a non-compliant fisher, the expected profit is the profit minus the fine μ multiplied by the intensity of control. The intensity of control is denoted as λ_1 when there is only one non-compliant fisher, and λ_2 when there are two non-compliant fishers, with $\lambda_2 > \lambda_1$. To follow the assumptions about the control function, we require that λ_1 and λ_2 lie between 0 and 1. The possible outcomes of the short-run game, when punishment is effective, are summarized in the proposition below, the proof of which is in Appendix B.

Proposition 2 (Harvesting game with quotas and punishment). *Assume* $1 > \lambda_2 > \lambda_1 > 0$ *and* $\mu > 0$. *Set*

$$\begin{aligned} \mu_1 &:= \pi\left(q^{CN}(X), q^Q(X); X\right) - \pi\left(q^Q(X), q^Q(X); X\right); \\ \mu_2 &:= \pi\left(q^{CN}(X), q^{CN}(X); X\right) - \pi\left(q^Q(X), q^{CN}(X); X\right) \end{aligned} \tag{6}$$

[16] Also known as anti-dilemma game, harmony game or efficient dominant-strategy game, a prisoner's delight game is further characterized by the condition $\pi^e\left(q^{CN}(X), q^{CN}(X); X\right) > \pi^e\left(q^{CN}(X), q^Q(X); X\right)$ which cannot be met in our setup. Nevertheless, we use the denomination prisoner's delight game as in a classical anti-coordination game there is a Pareto optimal solution that is also the unique Nash equilibrium of the game. See Wang and Yang (2003), for the classification of 2×2 matrix games.

and

$$\mu_3 := \pi\left(q^{CN}(X), q^{CN}(X); X\right) - \pi\left(q^{Q}(X), q^{Q}(X); X\right). \tag{7}$$

Moreover, define the constants $\tilde{\mu}_1 = \mu_1/\lambda_1$, $\tilde{\mu}_2 = \mu_2/\lambda_2$, $\tilde{\mu}_3^1 = \mu_3/\lambda_1$ *and* $\tilde{\mu}_3^2 = \mu_3/\lambda_2$. *The following statements hold:*

- *For* $\mu < \min\{\tilde{\mu}_1; \tilde{\mu}_2\}$, *the unique Nash-equilibrium is* $\left(q^{CN}(X), q^{CN}(X)\right)$ *and the game is a prisoner's delight for* $\mu < \tilde{\mu}_3^2$ *and a prisoner dilemma for* $\mu > \tilde{\mu}_3^2$;
- *For* $\tilde{\mu}_2 < \mu < \tilde{\mu}_1$, *the unique Nash-equilibria are* $\left(q^{Q}(X), q^{CN}(X)\right)$ *and* $\left(q^{CN}(X), q^{Q}(X)\right)$, *and the game is of anti-coordination;*
- *For* $\tilde{\mu}_1 < \mu < \tilde{\mu}_2$, *the unique Nash-equilibria are* $\left(q^{CN}(X), q^{CN}(X)\right)$ *and* $\left(q^{Q}(X), q^{Q}(X)\right)$, *and the game is of coordination;*
- *For* $\mu > \max\{\tilde{\mu}_1; \tilde{\mu}_2\}$, *the unique Nash-equilibrium is* $\left(q^{Q}(X), q^{Q}(X)\right)$ *and the game is a prisoner's delight for* $\mu > \tilde{\mu}_3^1$ *and a prisoner dilemma for* $\mu < \tilde{\mu}_3^1$.

Let us remark that it is always possible to increase the level of punishment so that the compliant harvesting strategy $\left(q^Q, q^Q\right)$ is the unique Nash equilibrium of the game. Specifically, the punishment should be such that $\mu > \mu^* := \max\{\tilde{\mu}_1; \tilde{\mu}_2\}$. However, if the control intensities λ_1 and λ_2 are low, the required punishment μ^* to ensure compliance may be extremely high and difficult for fishers to be accepted.

3.3 Certain Punishment (CATCH)

Assume that controls are *certain* and the amount of punishment is positive, i.e. $\mu > 0$. This case can be achieved if an IT control system such as CATCH is implemented in the fishery. In this case, the expected profit of a compliant fisher is the same as her actual profit. On the other hand, the expected profit of a non-compliant fisher is equal to the profit minus the certain fine, μ. The proposition below (the proof of which is in Appendix B) summarizes the possible outcomes of the short-run game when punishment is effective.

Proposition 3 (Harvesting game with quotas and certain punishment). *Assume* $\lambda = 1$ *and* $\mu > 0$. *Consider* μ_1, μ_2 *and* μ_3 *defined in* (6)-(7). *Then, the results of Proposition 2 hold once* $\tilde{\mu}_1$ *and* $\tilde{\mu}_2$ *are replaced with* μ_1 *and* μ_2, *respectively, and once* $\tilde{\mu}_3^1$ *and* $\tilde{\mu}_3^2$ *are both replaced with* μ_3.

In order for the compliant strategy $\left(q^Q(X), q^Q(X)\right)$ to be the unique Nash equilibrium of the game, the necessary level of punishment is $\mu > \mu^+ := \max\{\mu_1, \mu_2\}$. This means that the imposed fine should be slightly higher than the maximum profit gap a fisher can achieve by switching from the compliant to the non-compliant strategy. However, this fine may still be much lower than in the case without the CATCH system and may be considered reasonable and acceptable by most fishers.

So far, the investigation has been based on the assumption that the biomass level remains constant and is equal to X. Under this assumption, the CATCH mechanism, which ensures a probability of control and punishment independent of the level of harvesting, has the advantage that $\left(q^{CN}(X), q^{CN}(X)\right)$ (i.e., the non-compliant strategy) is never a Nash equilibrium of the game for a *reasonably* high fine. However, the biomass level X changes over time and the survival of the species can be threatened by a fixed mechanism of quotas even in the case of fishers who comply with the rules. This is demonstrated in the following, where standard logistic growth is assumed for the biomass, and the outcome under the compliant strategy, ensured for example by the CATCH system, is contrasted with that under the non-compliant strategy. Furthermore, we show that a certain punishment, ensured by the CATCH system with a constant fine, may also induce non-compliance when the biomass level evolves, suggesting the implementation of a punishment mechanism that depends on the levels of biomass and harvesting.

4 The Long-Run Harvesting Game with and without Quotas: The Case of Logistic Growth, Constant Price, and Quadratic Harvesting Costs

The investigation of the short-run game of Sect. 3 shows that the CATCH system, when combined with a specific punishment plan, can effectively ensure compliance with the quotas. The fine imposed is proportionate to the profit gap resulting from not complying to the quotas. However, it is important to conduct a long-term analysis to uncover potential risks associated with the quota system. Firstly, even compliance with the quota system may not prevent a situation of over-exploitation. Secondly, the equilibria resulting from sustainable harvesting through quota compliance may be unstable, making the system susceptible to disruptions in the biomass levels. These disruptions are often caused by external factors or shocks, such as disease outbreaks, seasonal changes in the ecosystem, and the invasion of predatory species.[17]

The fishing quota system poses certain dangers, which can be illustrated using a well-known dynamic framework for the evolution of biomass. In this model, we assume the resource undergoes logistic growth under harvesting, see, e.g., Gamito (1998). In discrete time, this dynamics can be written as[18]

$$X_{t+1} = X_t + rX_t \left(1 - \frac{X_t}{K}\right) - H\left(X_t\right) \tag{8}$$

where r is the specific growth rate, K is the carrying capacity of the species and $H\left(X\right)$ is the current harvesting, as specified below. For simplicity, we will omit the explicit dependence of biomass X on t when not strictly necessary.

According to the fishery model developed earlier, we consider the actions of two fishers. Moreover, landing costs are quadratic and biomass-dependent with the following specification (Smith 1969)[19]

[17] The 2002 reform of the EU's Common Fisheries Policy Commission of the European Communities (2009) stated that the exploitation of aquatic living resources should ensure economic, environmental and social sustainability. Sustainability means that *the exploitation of a stock should be done in such a way that the future exploitation of the stock will not be prejudiced and that it does not harm the marine eco-systems*, Commission of the European Communities (2002).

[18] This natural growth function is commonly used to describe stock growth in fisheries, see, e.g., Clark (1990) and Nostbakken (2013), Bischi et al. (2015).

[19] This cost function is convex and increasing in extraction, while it is decreasing in stock size. This quadratic cost structure is commonly used to represent expenses in models of renewable resources, where the cost of

$$c\left(q_i; X\right) = \frac{\psi q_i^2}{X} \tag{9}$$

where q_i represents the amount of a specific species of fish caught by fisher i and wholly supplied to the market. The constant ψ is a *technological coefficient*, the higher ψ is the higher the cost of harvesting for a given amount of biomass. This formulation for the cost reflects the impact of congestion and gear saturation issues on the fishery production function.

To complete the economic setup, we consider a fishery market characterized by a constant price a as in Nostbakken (2013). The assumption of a fixed price is commonly supported by the availability of numerous substitutes for each fish species, and the fact that fish is considered a staple food for most consumers. The profit of fisher i is then given by

$$\pi\left(q_i, q_{-i}; X\right) = a q_i - \frac{\psi q_i^2}{X}. \tag{10}$$

Without fishing quotas imposed by a regulator, fishers maximize myopically their current profit as common in open access fisheries, see Hardin (1968) and Quaas and Skonhoft (2022). Hence, their level of harvesting solves the following maximization problem:

$$\underset{q_i \in [0, X - q_{-i}]}{\operatorname{argmax}} \; \pi\left(q_i, q_{-i}; X\right) \tag{11}$$

that has solution

$$BR_i\left(q_{-i}; X\right) = \begin{cases} q^{CN}\left(X\right) := \frac{aX}{2\psi} & \text{if} \quad \frac{aX}{2\psi} < X - q_{-i} \\ X - q_{-i} & \text{if} \quad \frac{aX}{2\psi} > X - q_{-i} \end{cases}. \tag{12}$$

Moreover, we assume that fishers act as Nash players, therefore at each time t their harvesting solves $q_1 = BR_1\left(q_2; X\right)$ and $q_2 = BR_2\left(q_1; X\right)$. Assuming $\psi > a (> 0)$, there is a unique solution $\left(q^{CN}\left(X\right), q^{CN}\left(X\right)\right)$ of the system.[20] This solution implies that without restrictions each fisher finds it convenient to harvest the Cournot-Nash equilibrium harvesting $q^{CN}\left(X\right)$ gaining a profit

$$\pi\left(q^{CN}\left(X\right), q^{CN}\left(X\right); X\right) := \frac{a^2 X}{4\psi} \tag{13}$$

and the total harvesting is

extraction increases as the resource becomes scarcer, see Clark (1990), Szidarovszki and Okuguchi (1998), Conrad and Smith (2012) and Nostbakken (2013). Bischi et al. (2013) present its derivation from a production function with fishing effort (labor) and fish biomass (capital) as production inputs.

[20] The condition $\psi > a$ is imposed to avoid the trivial and unrealistic case such that the Cournot-Nash strategy implies the harvesting of the entire biomass in a single period. In fact, violating this condition allows for multiple *Cournot-Nash equilibrium harvesting*, all of which imply the harvesting of the entire biomass in a single period.

$$H\left(X\right)=2q^{CN}\left(X\right)=\frac{aX}{\psi}. \tag{14}$$

To investigate the problem of over-exploitation caused by Cournot-Nash harvesting among *myopic* fishers in a dynamic setting, we will employ the concept of maximum sustainable catch or MSY. MSY is a widely used tool for managing fisheries and represents the level of current harvesting that maximizes total landing while preserving the existence of the fish species. The MSY with logistic growth can be calculated easily and is given by $H^{MSY}=\frac{Kr}{4}$, see Jensen (1975). *Sustainable harvest* is defined as the MSY, while any catch above the MSY is considered *over-exploitation*. It is evident that in a dynamic setting, Cournot-Nash harvesting among myopic fishers leads to over-exploitation when

$$X>\frac{r\psi}{4a}K. \tag{15}$$

To prevent over-exploitation, a regulator can enforce catch limits through TACs based on MSY and then allocate fishing quotas to individual exploiters. Specifically, the authority can set a maximum limit on the amount of resources each fisher can harvest (quota), which must not exceed a certain percentage, denoted by $\eta\in(0,1]$, of the MSY divided by the number of fishers, that is $Q=\frac{\eta Kr}{8}$. Hence, when a fisher complies with the quota, she harvests the quantity

$$q^{Q}\left(X\right)=\min\left\{\frac{aX}{2\psi},\frac{\eta Kr}{8}\right\}. \tag{16}$$

The individual profits of a fisher that harvests a fraction η of the MSY quota are given by

$$\pi\left(Q,Q;X\right)=\frac{\eta Kr}{8}\left(a-\frac{\eta Kr\psi}{8X}\right). \tag{17}$$

In the following, we compare the global dynamics of the fishery model under six different scenarios. Benchmarks without quotas: (i) *No harvesting*, $H\left(X\right)=0$, (ii) *(Partial) MSY harvesting*, $H\left(X\right)=\frac{\eta Kr}{4}$, (iii) *Cournot-Nash harvesting*, $H\left(X\right)=2q^{CN}\left(X\right)$. Benchmarks with quotas: (iv) *Full compliance at MSY*, $H\left(X\right)=2q^{Q}\left(X\right)$, (v) *Partial compliance at MSY*, (vi) *Full compliance at partial MSY*. The last three scenarios require enforcement mechanisms that involve stringent controls by a supervisory authority. Moreover, the last scenario can be regarded as a robustness benchmark of the case (iv) and is reported in Appendix A for the sake of space.

This comparison highlights several risks associated to these harvesting strategies. Cournot-Nash harvesting leads to over-exploitation and reduces the species' resilience. On the other hand, MSY prevents over-exploitation but diminishes resilience against negative biomass level fluctuations. Finally, the fishing quota system does not completely solve over-exploitation issues but it enhances the species' resilience capabilities.

We begin by examining the first three scenarios, which are regarded as benchmark cases. Subsequently, we analyze harvesting compliance within a fishing quota system and compare the overall dynamics of this model with the three benchmarks.

4.1 Benchmarks Without Quotas

Here let us focus on three benchmark cases without quotas of the dynamic model described in Eq. (8), which illustrate the dynamics of the fishery. In the first benchmark, we examine the case where there is *no harvesting*. In the second benchmark, harvesting is defined as a fraction $\eta \in (0, 1]$, of the maximum sustainable catch so that in (8) harvesting is represented by $H(X) = \frac{\eta K r}{4}$. We refer to this case as *partial MSY harvesting* when $\eta \in (0, 1)$ and *MSY harvesting* when $\eta = 1$. In the third benchmark, *Cournot-Nash harvesting*, each agent harvests at the Cournot-Nash level, meaning $H(X) = \frac{aX}{\psi}$, where $a < \psi$ to ensure that $H(X) < X$.

In the next proposition (the proof of which is in Appendix B), we review the possible equilibria, their stability properties and the global dynamics for these three benchmark cases.

Proposition 4 (Dynamics without quotas). *Assume $r, K > 0$, $\psi > a > 0$ and $\eta \in (0, 1]$. In the case of:*

(i) ***No harvesting*** $(H(X) = 0)$, *the equilibria of* (8) *are $\bar{X}_1^N = 0$ and the carrying capacity $\bar{X}_2^N = K$.*

- *The extinction equilibrium $\bar{X}_1^N$ is always unstable;*
- *For $0 < r < 2$, the carrying capacity equilibrium $\bar{X}_2^N$ is asymptotically stable with basin of attraction given by $\mathcal{B}\left(\bar{X}_2^N\right) = \left(0, \frac{1+r}{r} K\right)$;*
- *At $r{=}2$, $\bar{X}_2^N$ loses stability through a flip bifurcation, after which a period-doubling regime conjugated with the standard logistic map occurs, that is, orbits in $\left(0, \frac{1+r}{r} K\right)$ are attracted to a 2^k-cycle for $2 < r \lessapprox 2.56994$ while they are chaotic for $2.56994 \lessapprox r \leq 3$;*
- *For $r{>}3$, resource extinction occurs in finite time.*

(ii) ***(Partial) MSY harvesting*** $(H(X) = \frac{\eta K r}{4})$, *the equilibria of* (8) *are*

$$\bar{X}_{1,2}^{MSY} = \frac{K}{2}\left(1 \mp \sqrt{1-\eta}\right) > 0. \tag{18}$$

- *For $\eta \in (0, 1)$, $\bar{X}_1^{MSY}$ is always unstable, while $\bar{X}_2^{MSY}$ is asymptotically stable with basin of attraction given by $\mathcal{B}\left(\bar{X}_2^{MSY}\right) = \left(\bar{X}_1^{MSY}, K\frac{2+r\left(1+\sqrt{1-\eta}\right)}{2r}\right)$ for $0 < r < \frac{2}{\sqrt{1-\eta}}$; at $r = \frac{2}{\sqrt{1-\eta}}$, $\bar{X}_2^{MSY}$ loses stability through a flip bifurcation, after which a period-doubling regime conjugated with the standard logistic map occurs, that is, orbits in $\left(\bar{X}_1^{MSY}, K\frac{2+r\left(1+\sqrt{1-\eta}\right)}{2r}\right)$ are attracted to a 2^k-cycle for $\frac{2}{\sqrt{1-\eta}} < r \lessapprox \frac{2.56994}{\sqrt{1-\eta}}$ while they are chaotic for $\frac{2.56994}{\sqrt{1-\eta}} \lessapprox r \leq \frac{3}{\sqrt{1-\eta}}$, finally for $r > \frac{3}{\sqrt{1-\eta}}$ resource extinction occurs in finite time;*

- *For* $\eta = 1$, $\bar{X}_{1,2}^{MSY} = \bar{X}^{MSY} = \frac{K}{2}$; $\frac{K}{2}$ *is a semi-asymptotically stable (unstable on the left-hand side and asymptotically stable on the right-hand side) equilibrium with basin of attraction* $\mathcal{B}\left(\frac{K}{2}\right) = \left[\frac{K}{2}, \frac{K(2+r)}{2r}\right]$*. All trajectories starting with initial conditions outside* $\mathcal{B}\left(\frac{K}{2}\right)$ *lead to extinction in finite time.*

(iii) ***Cournot-Nash harvesting*** $\left(H(X) = \frac{aX}{\psi}\right)$:

- *For* $0 < r < \frac{a}{\psi}$, *the extinction equilibrium* $\bar{X}_1^{CN} = 0$ *is the unique equilibrium in* $[0, +\infty)$ *and is asymptotically stable;*
- *At* $r = \frac{a}{\psi}$, *a transcritical bifurcation occurs at which the extinction equilibrium* $\bar{X}_1^{CN}$ *merges with the equilibrium*

$$\bar{X}_2^{CN} = K\left(1 - \frac{a}{r\psi}\right). \tag{19}$$

- *For* $\frac{a}{\psi} < r < 2 + \frac{a}{\psi}$, $\bar{X}_1^{CN}$ *is unstable,* $\bar{X}_2^{CN}$ *is asymptotically stable with basin of attraction given by* $\mathcal{B}\left(\bar{X}_2^{CN}\right) = \left(0, K\frac{1+r-\frac{a}{\psi}}{r}\right)$;
- *At* $r = 2 + \frac{a}{\psi}$, $\bar{X}_2^{CN}$ *loses stability through a flip bifurcation, after which a period-doubling regime conjugated with the standard logistic map occurs, that is, orbits in* $\left(0, K\frac{1+r-\frac{a}{\psi}}{r}\right)$ *are attracted to a* 2^k*-cycle for* $2 + \frac{a}{\psi} < r \lessapprox \frac{a}{\psi} + 2.56994$ *while they are chaotic for* $\frac{a}{\psi} + 2.56994 \lessapprox r \leq \frac{a}{\psi} + 3$;
- *For* $r > 3 + \frac{a}{\psi}$, *resource extinction occurs in finite tim*e.

Based on the global analysis of the three benchmark models, we observe that a high growth rate results in extinction in all cases except under MSY. When there is no harvesting, extinction due to resource overcrowding occurs when the growth rate exceeds the threshold $r=3$. In the case of partial MSY harvesting, extinction occurs when the growth rate exceeds the threshold $r = \frac{3}{\sqrt{1-\eta}} > 3$. For Cournot-Nash harvesting, extinction occurs for growth rates above the threshold $r = 3 + \frac{a}{\psi} > 3$. Therefore, harvesting helps alleviate the issue of overcrowding and harvesting based on MSY prevents extinction by overcrowding. Additionally, MSY and partial MSY do not lead to extinction due to the low reproduction rate of the species of fish. On the other hand, Cournot-Nash harvesting can cause extinction and an extinction lower-bound threshold of r exists and is given by the ratio a/ψ.

Analyzing the equilibria of the benchmark models, we observe that the equilibrium levels under MSY depend solely on biological parameters such as the growth rate (r) and the carrying capacity (K). In contrast, the equilibrium levels of biomass in the case of Cournot-Nash harvesting also depend on the selling price (a) and on the technological coefficient of the cost function of harvesting (ψ). Another important distinction between MSY and Cournot-Nash harvesting is the issue of over-exploitation. Under favorable economic conditions ($a > r\psi$), Cournot-Nash harvesting leads to the depletion of biomass and its extinction. On the other hand, MSY always ensures a non-extinction equilibrium for the fish species. Conversely, under negative economic conditions ($a < r\psi$), the Cournot-Nash strategy implies a lower level of exploitation of the fish species compared to the MSY strategy. In this case, the Cournot-Nash strategy is advantageous and preferable for the survival and resilience of the species.

The next proposition clarifies the existence of a threshold price $\tilde{a}$ such that for a price below the threshold, the biomass equilibrium under Cournot-Nash harvesting is higher than the biomass equilibrium under MSY harvesting, and vice versa when the price is above the threshold.

Corollary 1 *Consider biomass equilibria under (partial) MSY harvesting,* $\bar{X}_2^{MSY}$*, and Cournot-Nash harvesting,* $\bar{X}_2^{CN}$*, with* $a < r\psi$ *so that* $\bar{X}_2^{CN} > 0$*. Define the threshold price* $\tilde{a} = \frac{r\psi}{2}\left(1-\sqrt{1-\eta}\right)$.

- *If* $0 < a < \tilde{a}$*, then* $\bar{X}_2^{CN} > \bar{X}_2^{MSY}$ *and* $\mathcal{B}\left(\bar{X}_2^{CN}\right) \supset \mathcal{B}\left(\bar{X}_2^{MSY}\right)$.
- *If* $\tilde{a} < a < r\psi$*, then* $\bar{X}_2^{CN} < \bar{X}_2^{MSY}$ *and neither* $\mathcal{B}\left(\bar{X}_2^{CN}\right) \supset \mathcal{B}\left(\bar{X}_2^{MSY}\right)$ *nor* $\mathcal{B}\left(\bar{X}_2^{CN}\right) \subset \mathcal{B}\left(\bar{X}_2^{MSY}\right)$.

In Corollary 1, we found that when the selling price is within the range $(0 <)a \leq \tilde{a}$, the Cournot-Nash harvesting does not lead to over-exploitation and helps the fish species maintain better resilience. In this scenario, a fishing quota system is not needed. However, when $\tilde{a} < a < r\psi$, the Cournot-Nash harvesting results in a reduction of the equilibrium level of biomass, indicating over-exploitation. It also increases the vulnerability of the fish population to overcrowding, potentially leading to extinction. This vulnerability can be quantified in terms of biomass levels in the range $\left(K\frac{1+r-\frac{a}{\psi}}{r}, K\frac{2+r\left(1+\sqrt{1-\eta}\right)}{2r}\right)$. Despite these drawbacks, this method provides greater resilience for the fish species in the event of shocks that reduce the ecosystem's fish population.

We are now ready to analyze the case where there is a monitoring system to track the amount of fish caught in the fishery and the agents respect the quotas imposed by the regulator.

4.2 Fishing Quota System: (iv) Full Compliance at MSY

Let us consider a scenario where fishers always adhere to a set of fixed fishing quotas based on MSY. This system emulates a fishery where a comprehensive tool, like the CATCH system, makes it unavoidable to dodge controls, and severe penalties make the non-compliant approach unprofitable. When measuring the resilience of the fishery in terms of the stability of non-extinction outcomes, this fishery policy makes the resource more resilient compared to Cournot-Nash harvesting or MSY harvesting quotas in each period. However, there are still drawbacks to this policy. For instance, there could be a stable non-extinction equilibrium that the fishery asymptotically approaches. Additionally, there could be a stable equilibrium where the fish species faces depletion and over-exploitation. According to Corollary 1, the latter scenario might occur when $r < \frac{2a}{\psi\left(1+\sqrt{1-\eta}\right)}$.

Following fishing quotas and guided by the principle of economic convenience, agents choose the Cournot-Nash harvesting level $q^{CN}(X) = \frac{aX}{2\psi}$ as long as it is below the allocated quota $Q = \frac{\eta K r}{8}$, and they harvest their own quota otherwise. Here we assume that agents comply with the quota because of the strict enforcement policies implemented by the authority.[21] The resulting total harvesting is given by

$$H\left(X\right)=\begin{cases}\frac{aX}{\psi} & \text{if } 0\leqslant X<\tilde{X}=\frac{\eta Kr\psi}{4a}\\ \frac{\eta Kr}{4} & \text{if } X>\tilde{X}=\frac{\eta Kr\psi}{4a}\end{cases}. \tag{20}$$

The map modeling the resource growth in compliance with the fishing quota system is then the following:

$$X_{t+1}=X_t+rX_t\left(1-\frac{X_t}{K}\right)-\min\left\{\frac{aX_t}{\psi};\frac{\eta Kr}{4}\right\}. \tag{21}$$

Notice that the map in (21) is *piecewise smooth*, as it changes definition for the biomass level

$$\tilde{X}=\frac{\eta Kr\psi}{4a}, \tag{22}$$

at which each fisher catches the same quantity under Cournot-Nash harvesting or under the harvesting quota, that is $q^{CN}=Q$. In the following, we refer to $\tilde{X}$ as the *kink* point of the map, see Bischi et al. (2014), Radi and Gardini (2015) and reference therein. In the next proposition (the proof of which is in Appendix B), we characterize the dynamics of the map (21) when the quota set by the regulator is equal to the MSY harvesting.

Proposition 5 (Full compliance with quotas at MSY). *Consider the fishery growth map with harvesting at fishing quotas as in (21), with $0<a<\psi$ and $\eta=1$. Consider $\bar{X}_{1,2}^{CN}$ and $\bar{X}^{MSY}$ defined in Proposition 4. The following dynamic scenarios occur:*

(A) *For $0<r<\frac{a}{\psi}$, the extinction equilibrium $\bar{X}_1^{CN}$ is asymptotically stable, $\bar{X}_2^{CN}$ is not an equilibrium of the model, while the MSY equilibrium $\bar{X}^{MSY}$ is semi-asymptotically stable (unstable on the left-hand side and asymptotically stable on the right-hand side) with basin of attraction $\mathcal{B}\left(\bar{X}^{MSY}\right)=\left[\bar{X}^{MSY},\frac{K(2+r)}{2r}\right]$, all the other orbits converge to $\bar{X}_1^{CN}$;*

(B) *At $r=\frac{a}{\psi}$, a transcritical bifurcation occurs at which $\bar{X}_1^{CN}$ and $\bar{X}_2^{CN}$ merge, the global dynamics of the model is as in point (i);*

(C) *For $\frac{a}{\psi}<r<\frac{2a}{\psi}$, $\bar{X}_1^{CN}$, $\bar{X}_2^{CN}$ and $\bar{X}^{MSY}$ are equilibria of the model, $\bar{X}_1^{CN}$ is unstable, $\bar{X}_2^{CN}$ is asymptotically stable with basin of attraction $\mathcal{B}\left(\bar{X}_2^{CN}\right)=\left(0,\bar{X}^{MSY}\right)\cup\left(\frac{K(2+r)}{2r},\frac{K\left(1+r+\sqrt{1+2r}\right)}{2r}\right)$, $\bar{X}^{MSY}$ is semi-asymptotically stable (unstable on the left-hand side and asymptotically stable on the right-hand side), with basin of attraction $\mathcal{B}\left(\bar{X}^{MSY}\right)=\left[\bar{X}^{MSY},\frac{K(2+r)}{2r}\right]$;*

[21] Compliance with the fishing quota system is taken for granted in this type of fishing. However, it is difficult to guarantee with a constant fine scheme even when using a CATCH system for controls. This aspect will be discussed later.

(D) *At* $r = \frac{2a}{\psi}$, *equilibrium* $\bar{X}^{MSY}$ *merges with* $\bar{X}_2^{CN}$ *and both collides with the kink point* $\tilde{X}$, *otherwise the dynamics is as in (iii);*

(E) *For* $\frac{2a}{\psi} < r < 2 + \frac{a}{\psi}$, $\bar{X}_1^{CN}$ *is unstable,* $\bar{X}_2^{CN}$ *is asymptotically stable, there are no other equilibria and*

$$\mathcal{B}\left(\bar{X}_2^{CN}\right) = \left(0, \max\left\{K\frac{1+r-\frac{a}{\psi}}{r}; \frac{K\left(1+r+\sqrt{1+2r}\right)}{2r}\right\}\right), \tag{23}$$

specifically, for $r \geq \frac{2a}{\psi} + 2\sqrt{\frac{a}{\psi}}$, *we have* $\mathcal{B}\left(\bar{X}_2^{CN}\right) = \left(0, K\frac{1+r-\frac{a}{\psi}}{r}\right)$;

(F) *At* $r = 2 + \frac{a}{\psi}$, *equilibrium* $\bar{X}_1^{CN}$ *loses stability through a flip (period-doubling) bifurcation;*

(G) *For* $2 + \frac{a}{\psi} < r \leq 3 + \frac{a}{\psi}$, *equilibria* $\bar{X}_{1,2}^{CN}$ *are unstable and orbits in* $\left(0, \max\left\{K\frac{1+r-\frac{a}{\psi}}{r}; \frac{K\left(1+r+\sqrt{1+2r}\right)}{2r}\right\}\right)$ *are attracted by either periodic or chaotic attractors that for* $r \geq \frac{2a}{\psi} + 2\sqrt{\frac{a}{\psi}}$ *are those of map* (8) *with Cournot-Nash harvesting;*

(H) *For* $r > 3 + \frac{a}{\psi}$, *resource extinction occurs in finite time.*

To further illustrate the implications of the results in Proposition 5, we present some numerical examples. We will not consider cases (B), (D) and (F) as they are bifurcation points, or cases (E) and (G) where there is no overexploitation at the equilibrium $\bar{X}_2^{CN}$, because $\bar{X}_2^{CN} > \bar{X}^{MSY}$ and a fishing quota system can only increase its basin of attraction. We also omit case (H) because extinction is the only possible outcome regardless of the fishing quota system. Instead, we focus on cases (A) and (C) of the previous proposition.

Case (A) indicates that a fishing quota system does not prevent the risk of extinction even in the case of full compliance. A numerical example illustrating this scenario is shown in Fig. 1. Panel (a) displays the case of constant harvesting equal to the MSY, and the staircase diagrams show that orbits leading to extinction in finite time exist. Panel (b) illustrates the case of Cournot-Nash harvesting, with all orbits leading to extinction in finite time. Panel (c) depicts the fishing quota system under full compliance, where the equilibrium of MSY and the extinction equilibrium coexist and are stable. The initial conditions that lead to extinction in finite time in Panel (a) also do so in Panel (c). All the other initial conditions lead to the equilibrium of MSY. This numerical example emphasizes that the fishing quota system does not eliminate the risk of extinction even in the case of full compliance. The fishery is particularly susceptible to external shocks. Any negative shock that reduces the equilibrium value of the fish species causes extinction in finite time. This is due to the semi-stability of the equilibrium of MSY (unstable on the left-hand side and asymptotically stable on the right-hand side).

Case (C) indicates that a fishing quota system may not prevent the risk of overexploitation even when full compliance is achieved. A numerical example showing this phenomenon is reported in Fig. 2. Panel (a) depicts a situation where the harvesting remains constant at the MSY, and the staircase diagram indicates that there exist orbits leading to extinction in finite time. Panel (b) shows the case of Cournot-Nash harvesting with an equilibrium resulting in fish stock levels considerably lower than those observed at the MSY equilibrium. In this

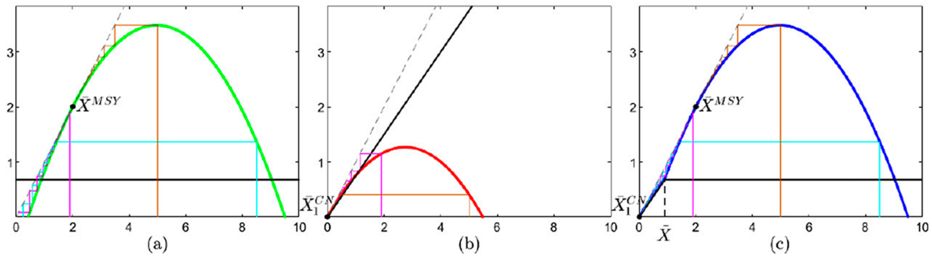

Fig. 1 Growth under map (8) with harvesting: Panel (**a**) at MSY; Panel (**b**) at Cournot-Nash level; Panel (**c**) in compliance with fishing quota at MSY. The dashed line represents biomass X, and the solid black line is harvesting. Staircase diagram of the trajectory starting at $X_0 = 1.9$ is in magenta. Staircase diagram of the trajectory starting at X_0=5 is in brown. Staircase diagram of the trajectory starting at $X_0 = 8.5$ is in cyan. Equilibria are intersections between the curve of growth and the dashed line and are spotted by a black dot. Parameters are a=0.9; ψ=1.2; $r = 0.9a/\psi$; K=4 and η=1

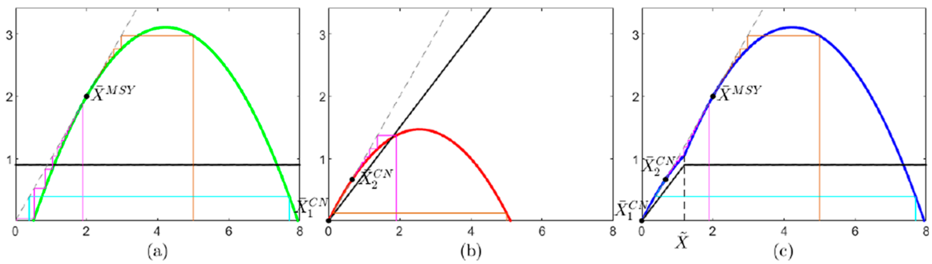

Fig. 2 Growth under map (8) with harvesting: panel (**a**) at MSY; panel (**b**) at Cournot-Nash level; Panel (**c**) in compliance with fishing quota at MSY. The dashed line represents biomass X, and the solid black line is harvesting. Staircase diagram of the trajectory starting at $X_0 = 1.9$ is in magenta. Staircase diagram of the trajectory starting at X_0=5 is in brown. Staircase diagram of the trajectory starting at $X_0 = 7.7$ is in cyan. Equilibria are intersections between the curve of growth and the dashed line and are spotted by a black dot. Parameters as in Fig. 1 but $r = 1.2a/\psi$

case, the equilibrium of over-exploitation is stable and prevents extinction. Finally, Panel (c) shows how a fishing quota system under full compliance can produce two stable equilibria. However, the equilibrium consistent with the MSY is only right-hand asymptotically stable: a negative shock reducing the level of fish in the MSY equilibrium implies convergence in finite time to the equilibrium with Cournot-Nash harvesting, causing a substantial depletion in the fish stock. In simple terms, overexploitation occurs.

These perils, related to a fixed fishing quota scheme, persist but are somewhat reduced imposing a quota that is a fraction of the MSY. For details, see Appendix 1.

4.3 Fishing Quota System: (v) Partial Compliance at MSY

Let us assume that any harvesting activity is controlled by the public authority (CATCH) so that a fixed fine μ is imposed with certainty whenever the harvesting level exceeds the quota. Additionally, let us assume that the two fishers are using Nash equilibrium strategies. Therefore, based on Proposition 3, we can state the harvesting as follows:

$$H\left(X\right)=\begin{cases}2q^{CN}\left(X\right) & \text{if } \mu<\min\left\{\mu_1;\mu_2\right\}\\ 2q^{Q}\left(X\right) & \text{if } \mu>\max\left\{\mu_1;\mu_2\right\}\\ q^{Q}\left(X\right)+q^{CN}\left(X\right) & \text{otherwise}\end{cases} \tag{24}$$

where μ_1 and μ_2 are defined in (6).

Indeed, for $\mu<\min\left\{\mu_1;\mu_2\right\}$, the unique Nash equilibrium is $\left(q^{CN}\left(X\right),q^{CN}\left(X\right)\right)$, hence the total harvesting is $2q^{CN}\left(X\right)$. For $\mu>\max\left\{\mu_1;\mu_2\right\}$, the unique Nash equilibrium is $\left(q^{Q}\left(X\right),q^{Q}\left(X\right)\right)$, hence the total harvesting is $2q^{Q}\left(X\right)$. In the other cases, either a non-coordination game takes place with Nash equilibria $\left(q^{CN}\left(X\right),q^{Q}\left(X\right)\right)$ and $\left(q^{Q}\left(X\right),q^{CN}\left(X\right)\right)$, hence the total harvesting is $q^{CN}\left(X\right)+q^{Q}\left(X\right)$, or the game is of coordination, with Nash equilibria $\left(q^{Q}\left(X\right),q^{Q}\left(X\right)\right)$ and $\left(q^{CN}\left(X\right),q^{CN}\left(X\right)\right)$. In this last case, the total harvesting depends on which one of the two Nash equilibria will be played. Here we assume that these two equilibria are played with probability $\frac{1}{2}$. Hence, the total harvesting is $q^{CN}\left(X\right)+q^{Q}\left(X\right)$.

By straightforward algebra and considerations, we obtain the analytical expression for the harvesting function (24), see Appendix C. This leads to the following map that models the resource growth when there is only partial compliance with the fishing quota system:

$$X_{t+1}=\begin{cases}X_t+rX_t\left(1-\frac{X_t}{K}\right)-\frac{\eta Kr}{4} & \text{if } \tilde{X}<X_t<X^{+}\\ X_t+rX_t\left(1-\frac{X_t}{K}\right)-\frac{aX_t}{\psi} & \text{otherwise}\end{cases}, \tag{25}$$

where $\tilde{X}$ as in (22) and

$$X^{+}=\frac{\psi\left(aKr\eta+8\mu\right)+\psi\sqrt{\left(aKr\eta+8\mu\right)^2-a^2K^2r^2\eta^2}}{4a^2}. \tag{26}$$

The map in (25) not only admits the point of non-differentiability $\tilde{X}$, which is the kink point defined in (22), but also a point of discontinuity X^{+}, defined in (26). Moreover, the map (25) reduces to the map (8) with Cournot-Nash harvesting when $\mu=0$, see again Appendix C for details. In the next proposition (the proof of which is in Appendix B), we analyze the dynamics of (25) when a quota is set at the MSY. We focus on the equilibria, their stability and their basins of attraction for all possible values of the fine μ.

Proposition 6 (Partial compliance with quotas at MSY). *Consider the fishery growth map with partial compliance with the fishing quotas as in (25), with $0<a<\psi$ and $\eta=1$. Consider $\bar{X}_{1,2}^{CN}$ and $\bar{X}^{MSY}$ defined in Proposition 4. Define*

$$\tilde{\mu}:=\frac{K(2a-r\psi)^2}{32\psi}\ \textit{and}\ \hat{\mu}:=\min\left\{\frac{K\left(r^2\psi-2a(r+2)\right)^2}{32r\psi\left(2+r\right)};\frac{aK\left(2a\left(2+r\right)-r^2\psi\right)}{8r\psi}\right\}. \tag{27}$$

The following dynamic scenarios occur:

(A) *For* $0 < r < \frac{a}{\psi}$, *extinction equilibrium* $\bar{X}_1^{CN}$ *is asymptotically stable,* $\bar{X}_2^{CN}$ *is not an equilibrium of the model, while*

- *If* $\mu < \tilde{\mu}$, *the MSY equilibrium* $\bar{X}^{MSY}$ *is not feasible, all the other orbits converge to* $\bar{X}_1^{CN}$;
- *If* $\mu \geq \tilde{\mu}$, *the MSY equilibrium* $\bar{X}^{MSY}$ *is semi-asymptotically stable (unstable on the left-hand side and asymptotically stable on the right-hand side) with basin of attraction that is either* $\mathcal{B}\left(\bar{X}^{MSY}\right) = \left[\bar{X}^{MSY}, \frac{K(2+r)}{2r}\right]$ *when* $\mu > \hat{\mu}$, *or a subset of* $\left[\bar{X}^{MSY}, \frac{K(2+r)}{2r}\right]$ *when* $\mu < \hat{\mu}$, *and all the other orbits converge to* $\bar{X}_1^{CN}$;

(B) *At* $r = \frac{a}{\psi}$, *a transcritical bifurcation occurs at which* $\bar{X}_1^{CN}$ *and* $\bar{X}_2^{CN}$ *merge, the global dynamics of the model is as in point (i);*

(C) *For* $\frac{a}{\psi} < r < \frac{2a}{\psi}$, $\bar{X}_1^{CN}$, $\bar{X}_2^{CN}$ *and* $\bar{X}^{MSY}$ *are equilibria of the model,* $\bar{X}_1^{CN}$ *is unstable,* $\bar{X}^{MSY}$ *is semi-asymptotically stable (unstable on the left-hand side and asymptotically stable on the right-hand side) with basin of attraction that is either* $\mathcal{B}\left(\bar{X}^{MSY}\right) = \left[\bar{X}^{MSY}, \frac{K(2+r)}{2r}\right]$ *when* $\mu > \hat{\mu}$, *or a subset of* $\left[\bar{X}^{MSY}, \frac{K(2+r)}{2r}\right]$ *when* $\mu < \hat{\mu}$, $\bar{X}_2^{CN}$ *is asymptotically stable;*

(D) *At* $r = \frac{2a}{\psi}$, *equilibrium* $\bar{X}^{MSY}$ *merges with* $\bar{X}_2^{CN}$ *and both equilibria collide with the kink point* $\tilde{X}$ *when* $\mu > \hat{\mu}$; *otherwise the dynamics is as in (iii)*;

(E) *For* $\frac{2a}{\psi} < r < 2 + \frac{a}{\psi}$, $\bar{X}_1^{CN}$ *is unstable,* $\bar{X}_2^{CN}$ *is asymptotically stable and there are no other equilibria and, for* $r \geq \frac{2a}{\psi} + 2\sqrt{\frac{a}{\psi}}$; *moreover, it is* $\mathcal{B}\left(\bar{X}_2^{CN}\right) = \left(0, K\frac{1+r-\frac{a}{\psi}}{r}\right)$;

(F) *At* $r = 2 + \frac{a}{\psi}$, *equilibrium* $\bar{X}_1^{CN}$ *loses stability through a flip (period-doubling) bifurcation;*

(G) *For* $2 + \frac{a}{\psi} < r \leq 3 + \frac{a}{\psi}$, *equilibria* $\bar{X}_{1,2}^{CN}$ *are unstable and orbits in* $\left(0, \max\left\{K\frac{1+r-\frac{a}{\psi}}{r}; \min\left\{X^+, \frac{K\left(1+r+\sqrt{1+2r}\right)}{2r}\right\}\right\}\right)$ *are attracted by either periodic or chaotic attractors that for* $r \geq \frac{2a}{\psi} + 2\sqrt{\frac{a}{\psi}}$ *are those of map* (8) *with Cournot-Nash harvesting;*

(H) *For* $r > 3 + \frac{a}{\psi}$, *resource extinction occurs in finite time.*

The results in Proposition 6 indicate that a low fine μ can have four possible effects. First, it can reduce the basin of attraction of the MSY equilibrium. Second, it can render the MSY equilibrium unfeasible. Third, it can expand the basin of attraction of the equilibria of overexploitation. Fourth, it can broaden the set of orbits that converge to extinction in finite time. The specific outcome among these four scenarios depends on the values of the other parameters.

These perils of the fishing quota system with fixed fines and partial compliance are illustrated in the following numerical simulations. Figure 3 shows the dynamics of map (25) under three different levels of fine. The parameters are chosen such that the MSY equilibrium $\bar{X}^{MSY}$ exists and is stable for $\mu > \tilde{\mu}$ and does not exist for $\mu < \tilde{\mu}$. There is only one other unique stable equilibrium, that is the extinction equilibrium. Specifically, we are in the parameter region $r \leq a/\psi$, that is case (A) of Proposition 6. We know that under full compliance, the basin of attraction of the MSY equilibrium is $\mathcal{B}\left(\bar{X}^{MSY}\right)=\left[\bar{X}^{MSY},\frac{K(2+r)}{2r}\right]$, see Proposition 5. All the other orbits lead to extinction in finite time. If the fine μ is sufficiently high, e.g. $\mu=1.5$, the basin of attraction of equilibrium $\bar{X}^{MSY}$ remains the same and all other orbits imply extinction in finite time, see Fig. 3(a). However, reducing the fine μ to 0.2, we observe that the basin of attraction of the MSY equilibrium, $\bar{X}^{MSY}$, shrinks to $\mathcal{B}\left(\bar{X}^{MSY}\right)=\left[\bar{X}^{MSY},X^{+}\right]$, see Fig. 3(b). Further reducing μ from 0.2 to 0.05, we observe that the MSY equilibrium disappears and all orbits converge to the equilibrium of extinction $\bar{X}_1^{CN}$, see Fig. 3(c). Hence, even under the certainty of punishment, avoiding extinction depends on the imposed fine.

The numerical example in Fig. 4 illustrates a situation where $a/\psi<r<2a/\psi$, which corresponds to case (C) of Proposition 6. When $\mu>\tilde{\mu}$, both the MSY equilibrium, $\bar{X}^{MSY}$, and the equilibrium of overexploitation, $\bar{X}_2^{CN}$, are stable while the equilibrium of extinction is unstable. However, when $\mu<\tilde{\mu}$ the equilibrium of overexploitation $\bar{X}_2^{CN}$ is the only stable equilibrium and the MSY equilibrium does not exist. Moreover, we know that under full compliance, the basin of attraction of the equilibrium of MSY is $\mathcal{B}\left(\bar{X}^{MSY}\right)=\left[\bar{X}^{MSY},\frac{K(2+r)}{2r}\right]$, see Proposition 5. The set $\mathcal{B}\left(\bar{X}_2^{CN}\right)=\left(0,\bar{X}^{MSY}\right)\cup\left(\frac{K(2+r)}{2r},\frac{K\left(1+r+\sqrt{1+2r}\right)}{2r}\right)$ is instead the basin of attraction of the equilibrium of overexploitation. This is the scenario represented in Fig. 4(a), where $\mu=1.5>\hat{\mu}$.

When we decrease the fine μ from 1.5 to 0.2, corresponding to the case where $\tilde{\mu}<\mu<\hat{\mu}$, the basin of attraction of the MSY equilibrium $\bar{X}^{MSY}$ shrinks to $\mathcal{B}\left(\bar{X}^{MSY}\right)=\left[\bar{X}^{MSY},X^{+}\right]$ and the basin of attraction of the over-exploitation equilibrium also shrinks but includes points that before were part the basin of attraction of $\bar{X}^{MSY}$. Instead, the initial conditions leading to extinction in finite time expand. See Fig. 4(b). Further reducing μ from 0.2 to 0.05, which corresponds to the case $\mu<\hat{\mu}$, the MSY equilibrium is not feasible anymore, the equilibrium of overexploitation is stable and with basin given by $\mathcal{B}\left(\bar{X}_2^{CN}\right)=\left(0,K\frac{1+r-a/\psi}{r}\right)$. All the other orbits lead to extinction in finite time. See Fig. 4(c).

If $2a/\psi<r<2+a/\psi$, the equilibrium of MSY is not feasible and the trajectories either converge to the equilibrium $\bar{X}_2^{CN}$ or a periodic/chaotic attractor exists. For this parameters configuration, increasing the fine μ has the unique positive effect of increasing the basin of attraction of the equilibrium $\bar{X}_2^{CN}$ if stable or the basin of attraction of the periodic/chaotic

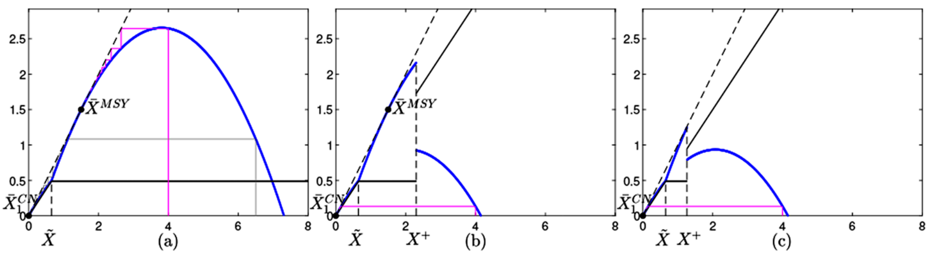

Fig. 3 In blue growth under map (25): Panel (**a**) $\mu=1.5$; Panel (**b**) $\mu=0.2$; Panel (**c**) $\mu=0.05$. The dashed line represents biomass X, and the solid black line is harvesting. Staircase diagram of the trajectory starting at $X_0=4$ is in magenta. Staircase diagram of the trajectory starting at $X_0=6.5$ is in gray. Equilibria are intersections between the curve of growth and the dashed line and are spotted by a black dot. Parameters are $a=0.9$; $\psi=1.2$; $r=a/\psi-0.1$; $K=3$ and $\eta=1$

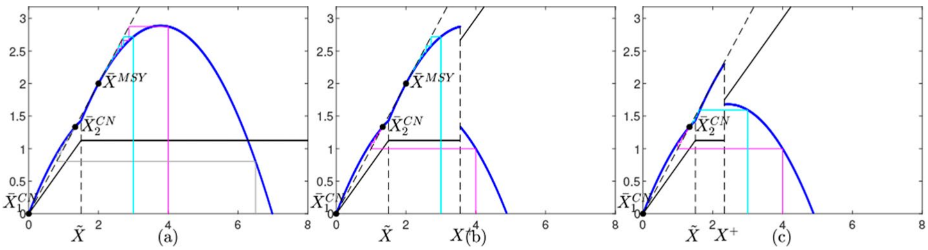

Fig. 4 In blue growth under map (25): Panel (**a**) $\mu=1.5$; Panel (**b**) $\mu=0.2$; Panel (**c**) $\mu=0.05$. The dashed line represents biomass X, and the solid black line is harvesting. Staircase diagram of trajectory starting at $X_0=4$ is in magenta. Staircase diagram of trajectory starting at $X_0=3$ is in cyan. Staircase diagram of trajectory starting at $X_0=6.5$ is in gray. Equilibria are intersections between the curve of growth and the dashed line and are spotted by a black dot. Parameters are $a=0.9$; $\psi=1.2$; $r=1.5a/\psi$; $K=4$ and $\eta=1$

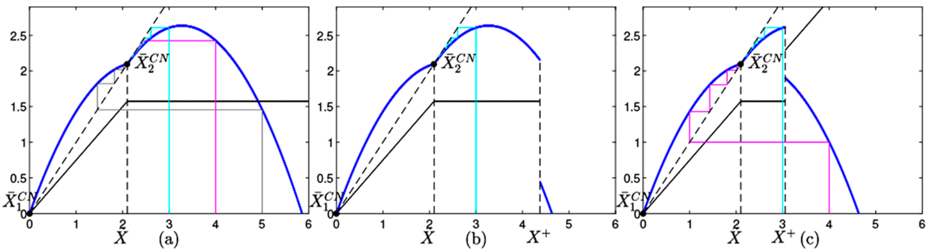

Fig. 5 In blue growth under map (25): panel (**a**) $\mu=1.5$; panel (**b**) $\mu=0.2$; panel (**c**) $\mu=0.05$. The dashed line represents biomass X, and the solid black line is harvesting. Staircase diagram of trajectory starting at $X_0=4$ is in magenta. Staircase diagram of trajectory starting at $X_0=3$ is in cyan. Staircase diagram of trajectory starting at $X_0=6.5$ is in gray. Equilibria are intersections between the curve of growth and the dashed line and are spotted by a black dot. Parameters are $a=0.9$; $\psi=1.2$; $r=2.1a/\psi-0.1$; $K=4$ and $\eta=1$

attractor otherwise. When we compare Fig. 5(a), obtained with $\mu=1.5>\hat{\mu}$, with Fig. 5(b), obtained with $\mu=0.2<\hat{\mu}$, and with Fig. 5(c), obtained with $\mu=0.05<\hat{\mu}$, we observe that in Fig. 5(a) the basin of attraction of the equilibrium $\bar{X}_2^{CN}$ is $\mathcal{B}\left(\bar{X}_2^{CN}\right)=\left(0,K\frac{2+r}{2r}\right)$, while in Fig. 5(b) and Fig. 5(c) this basin shrinks to $\mathcal{B}\left(\bar{X}_2^{CN}\right)=\left(0,K\frac{1+r-a/\psi}{r}\right)$. Let us underline that for $r>2a/\psi$, it holds that $\bar{X}_2^{CN}>\bar{X}^{MSY}$. Therefore, $\bar{X}_2^{CN}$ does not represent an equilibrium of over-exploitation in this last numerical example, and increasing the fine μ is beneficial, as it increases the basin of attraction of $\bar{X}_2^{CN}$.

In Fig. 6, we consider a scenario where reducing the fine μ only serves to increase the orbits that result in extinction in a finite time. The parameters satisfy the conditions of case (G) in Proposition 6. Consequently, the equilibrium points are $\bar{X}_{1,2}^{CN}$, both of which are unstable. We also have a stable 2-cycle around $\bar{X}_2^{CN}$. When $\mu=1.5$, the basin of attraction of this 2-cycle is $\left(0,\frac{K\left(1+r+\sqrt{1+2r}\right)}{2r}\right)$, as shown in Fig. 6(a). As we reduce the fine μ from 1.5 to 0.013, the basin of attraction of this stable 2-cycle shrinks to $(0,X^+)$, see Fig. 6(b). We observe that the cyan trajectory is not visible as it leads to extinction in a single period. Further reducing the fine from 0.013 to 0.008, the basin of attraction of the stable 2-cycle further shrinks to $\left(0, K\frac{1+r-\frac{a}{\psi}}{r}\right)$, see Fig. 6(c). We observe that neither the cyan nor the gray trajectories are visible as they lead to extinction in a single period.

4.4 A Comparison of the Benchmarks

To conclude, we compare all benchmarks through the bifurcation diagrams of Figs. 7 and 8. They show the long-term dynamics of the models for different values of the natural growth rate parameter r. Figure 7 depicts the benchmarks without quotas, that is, the cases of no harvesting (panel (a)), the case of partial MSY harvesting (panel (b)) and the case of Cournot-Nash harvesting (panel (c)). By comparing these three bifurcation diagrams, we can see that harvesting increases the risk of extinction. Specifically, for each level of the natural growth rate ($r < 1$), the basin of attraction for extinction (indicated in yellow), which represents the set of initial conditions that can lead to extinction, is broader under harvesting conditions. Notably, under Cournot-Nash harvesting, this basin is even wider than under partial Maximum Sustainable Yield (MSY) harvesting, particularly when η is set at, for example, 0.2. Furthermore, at low values of the natural growth rate ($r < 1$), Cournot-Nash harvesting results in the extinction of the resource, an outcome that could be avoided (at least for some initial conditions) under (partial) MSY harvesting.

Introducing a quota system with full compliance can significantly reduce the risk of extinction. Even when the natural growth rate ($r < 1$) is low, a stable equilibrium exists where the fish stock is not depleted, as illustrated by comparing Fig. 7(c) with Fig. 8(a). Additionally, when the natural growth rate r falls within the range of $(1,3)$, implementing a quota system enhances the basin of attraction for equilibria that prevent complete resource depletion, namely $\bar{X}^{MSY}$ (with basin in cyan) and $\bar{X}_2^{CN}$ (with basin in white).

However, for $r \in (1, 2)$, both equilibria $\bar{X}^{MSY}$ and $\bar{X}_2^{CN}$ coexist stably. Despite full compliance with the quota system, $\bar{X}_2^{CN}$ represents an equilibrium that can lead to resource overexploitation. In fact, the stock level at equilibrium $\bar{X}_2^{CN}$, corresponding to Cournot-Nash harvesting, is lower than that at equilibrium $\bar{X}^{MSY}$, which aligns with Maximum Sustainable Yield (MSY) harvesting. This indicates that the risk of overexploitation remains under harvesting conditions that comply with quotas; it becomes an issue related to the system's initial conditions.

Notably, an equilibrium consistent with Cournot-Nash levels appears for all $r>1$. For $r \in (1, 2)$, this equilibrium is stable and may suggest overexploitation of the resource. The

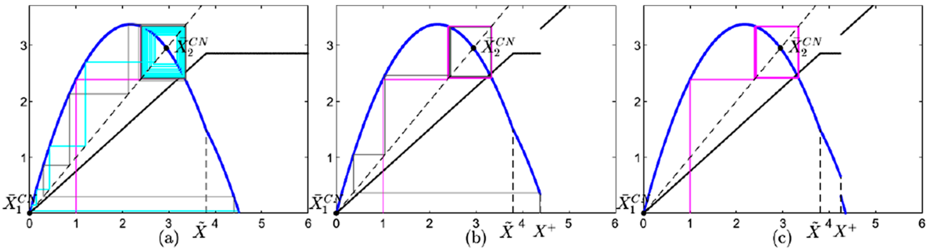

Fig. 6 In blue growth under map (25): Panel (**a**) $\mu=1.5$; Panel (**b**) $\mu=0.013$; Panel (**c**) $\mu=0.008$. The dashed line represents biomass X, and the solid black line is harvesting. Staircase diagram of trajectory starting at $X_0=1$ is in magenta. Staircase diagram of trajectory starting at $X_0 = 4.5$ is in cyan. Staircase diagram of trajectory starting at $X_0 = 4.35$ is in gray. Equilibria are intersections between the curve of growth and the dashed line and are spotted by a black dot. Parameters are $a=0.9$; $\psi=1.2$; $r = 2 + a/\psi + 0.1$; $K=4$ and $\eta=1$

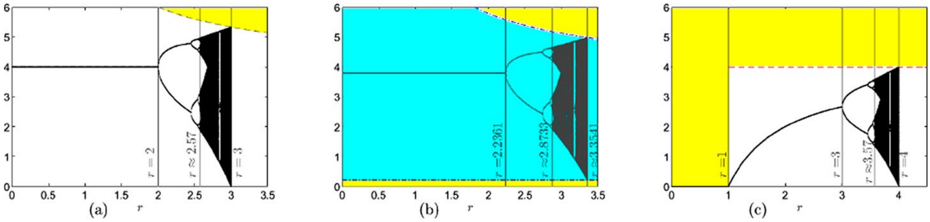

Fig. 7 Bifurcation diagrams with respect to the bifurcation parameter r. Panel (**a**): (**i**) No harvesting (long-run dynamics of X in black). Panel (**b**): (ii) (partial) MSY harvesting (long-run dynamics of X in gray). Panel (**c**): (iii) Cournot-Nash harvesting (long-run dynamics of X in black). Gray dashed curve $(X = (1+r)\,/\,(rK))$ marks the boundary of the basin of attraction of the equilibrium $\bar{X}_2^N$ or of another bounded attractor when $\bar{X}_2^N$ is unstable (panel (**a**)), blue dot-dashed curves $\left(X = K\left(1-\sqrt{1-\eta}\right)\right.$ and $\left.X = K\left(2+r\left(1+\sqrt{1-\eta}\right)\right)/\,(2r)\right)$ mark the borders of the basin of attraction of the equilibrium $\bar{X}_2^{MSY}$ or of another bounded attractor when $\bar{X}_2^{MSY}$ is unstable (panel (**b**)), red dashed curve $(X = K\,(1+r-a/\psi)\,/r)$ marks the border of the basin of attraction of the equilibrium $\bar{X}_2^{CN}$ or of another bounded attractor when $\bar{X}_2^{CN}$ is unstable (panel (**c**)). For a fixed value of r (in the horizontal axis), the vertical segments in yellow depict the points belonging to the basin of attraction of extinction (that is, $X_t \to 0$ or $X_t \to -\infty$ as $t \to +\infty$), in white (panel (**a**)) the basin of attraction of either equilibrium $\bar{X}_2^N$ or of the stable attractor originating when $\bar{X}_2^N$ is unstable, in cyan the basin of attraction of $\bar{X}_2^{MSY}$ and in white (panel (**c**)) the basin of attraction of either equilibrium $\bar{X}_2^{CN}$ or of the stable attractor originating when $\bar{X}_2^{CN}$ is unstable. Each panel also reports the bifurcation values of r as indicated in Proposition 4. Parameters: K=4; η=0.2; ψ=1 and a=1

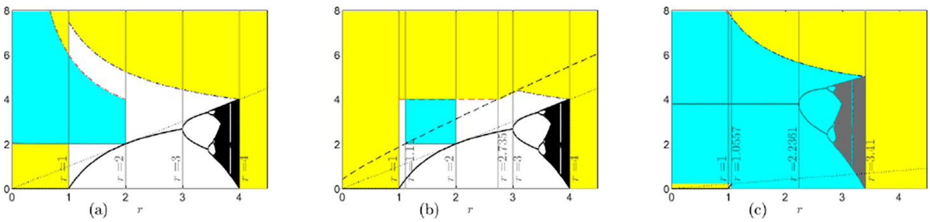

Fig. 8 Bifurcation diagrams with respect to the bifurcation parameter r. Panel (**a**): (iv) harvesting in compliance with quotas at MSY. Panel (**b**): (**v**) partial compliance at MSY. Panel (**c**): (vi) full compliance with quotas at partial MSY. Gray dashed curve is $X = X^+ = \left(\psi\,(aKr\eta + 8\mu) + \psi\sqrt{(aKr\eta+8\mu)^2 - a^2K^2r^2\eta^2}\right)/\left(4a^2\right)$, gray dotted curve is $X = \tilde{X} := (\eta K r \psi)\,/\,(4a)$, the red dot-dashed curve is $X = K\,(2+r)\,/\,(2r)$ (preimages of $\tilde{X}$). The blue dot-dashed and red dashed curves are defined in Fig. 7. For each value of r (in the horizontal axis), the yellow vertical segments are points that belong to the basin of attraction of extinction (see the caption of Fig. 7), the white vertical segments are the basin of attraction of either equilibrium $\bar{X}_2^{CN}$ or of the stable attractor originating when $\bar{X}_2^{CN}$ is unstable, the cyan vertical segment is the basin of attraction of either equilibrium $\bar{X}^{MSY}$ ($\bar{X}_2^{MSY}$, if η<1) or of the stable attractor originating when $\bar{X}^{MSY}$ is unstable. Each panel also reports the bifurcation values of r as indicated in Proposition 5 (panel (**a**)), in Proposition 6 (panel (**b**)), in Proposition 7 (panel (**c**)). Parameters as in Fig. 7

situation improves if quotas are set as a fraction of the MSY, as seen in Fig. 8(c). Conversely, the scenario deteriorates in the absence of full compliance, demonstrated in Fig. 8(b). Under such circumstances, extinction becomes certain for lower growth rates ($r < 1$), similar to when Cournot-Nash harvesting is applied. For higher values of the growth rate (r), there exists an equilibrium with MSY harvesting, but it coexists with the Cournot-Nash harvesting equilibrium. For $r > 2$, the situation resembles that of a fishery operated under Cournot-Nash harvesting; while the basin of attraction for extinction may diminish, this occurs only for specific values of r. For further details on these figures, please refer to the respective captions.

Summing up, the numerical examples show that:

- a digitized fishery control system can improve sustainability, but risks linked to improperly adjusted fishing quotas and enforcement remain;
- a quota system based on MSY does not prevent the risk of a resource becoming extinct over time, even under full compliance;
- the risk of extinction remains even if exploiters catch less than what the target imposes and choose a strategy to maximize their profits;
- non-adherence to quotas may occur due to low biomass, making harvesting unprofitable, or high biomass, where benefits of overfishing may exceed penalties. This poses a potential risk: if overfishing fines are not set appropriately, stock fluctuations can lead to overfishing and declines in resource stocks.

5 Conclusions

The use of new technologies that simplify assessments and verification of catch certificates for fishery products, such as the European fisheries certification program CATCH, represents a breakthrough for the auditing activity in the fishing industry. These technologies enable the control of fishing levels at their source without the need for expensive detection activities, which are often hindered by limited budgets made available to resource managers.

The introduction of new IT tools to enhance formal enforcement in the management of a fishing quota system is aimed at promoting sustainability. However, sustainability remains an open problem.

One potential threat to sustainability is linked to the TAC. Even with full compliance with the fishing quota regulations, a dynamic model suggests that over-exploitation may occur due to a fixed fishing quota established beforehand based on a TAC intended to achieve the MSY. The MSY equilibrium is not globally stable and may coexist with an equilibrium of over-exploitation or even lead to extinction in finite time. A potential solution could involve adaptively adjusting the TAC in response to changes in the biomass level over time. However, implementing this solution may pose challenges due to technical feasibility issues and associated costs. Estimating the MSY itself is already a complex task, see, e.g., Tsikliras and Froese (2019) and references therein. An alternative solution is to fix the TAC below MSY. However, not exploiting fisheries resources at MSY leads to the loss of production and rents from the fisheries and, as shown here, it only addresses the issue of overfishing partially.

An ineffective system of sanctions can hinder sustainability by failing to deter quota violations, even when the risk of punishment is certain. Even with strong enforcement, overexploitation can occur if the penalties do not account for the strong incentive to deviate when natural resources are abundant. Implementing a dynamic fine system could address this issue, but it may be challenging for resource managers to predict and account for the potential profits from overexploitation in advance.

In conclusion, an IT fishing control system addresses the challenge of enforcing controls and regulations through a comprehensive monitoring system. However, it does not resolve the issues of overfishing and illegal catch of fish from a stock that is well within safe biological limits. The findings from the dynamic model suggest potential policy indications to tackle these issues. These include implementing a system where both TAC and penalties are determined dynamically to anticipate fish depletion and fishers' profits.[22]

The findings from this study add to the existing economic literature on law enforcement in fisheries. However, the results are obtained using a basic, although largely adopted, biological model based on logistic growth. In our dynamic analysis, we focused on the primary benchmark cases that utilize deterministic enforcement mechanisms. This approach enabled us to thoroughly explore the implications and outcomes associated with this specific type of enforcement. Given the contextual framework of our analysis, we chose to set aside the case of probabilistic enforcement, which we intend to study separately in a future work. There are additional complexities to consider when dealing with a quota-regulated CPR with biologically interacting fish species. Management of many fisheries in the EU is based on assessing the stocks of individual species, even though most species are caught together with others. In multi-species assessments, the MSY reference point based on a single species often does not hold, and so the catch recommendations, see, e.g., Legović et al. (2010) and Guillen et al. (2013). In cases of predator-prey interaction, setting the maximum sustainable catch based on different MSYs for the prey and predator could lead to the extinction of the predator, as indicated in various studies, see, e.g., Kar and Legovic (2014) and references therein. These issues indicate the need for a more complex system to dynamically set quotas and punishments, taking into account biological interactions and their impact on total harvesting and profits.

Appendix A. A Robustness Check of Fishing Quota System: (vi) Full Compliance at Partial MSY

In this Appendix, we consider the resource growth model (21) in case of partial MSY harvesting, that is $0 < \eta < 1$. The aim is to investigate the global dynamics of this model to determine if setting a quota at a fraction of the MSY can solve some of the issues related to overexploitation and extinction that arise from fishing at the MSY. Similar to the approach in Sect. 4.2, we assume full compliance with the fishing quota system. We will begin by extending the results of Proposition 5 to the case of $\eta < 1$. The proof of the following proposition is in Appendix 2.

Proposition 7 (Full compliance with quotas at partial MSY). *Consider the fishery growth map with harvesting at fishing quotas as in* (21), *with* $0 < a < \psi$ *and* $0 < \eta < 1$. *Consider* $\bar{X}_{1,2}^{CN}$ *and* $\bar{X}_{1,2}^{MSY}$ *defined in Proposition* 4. *Define the following growth rate thresholds* $\tilde{r}_1$ *and* $\tilde{r}_2$:

$$\tilde{r}_1 := \frac{2a\left(1-\sqrt{1-\eta}\right)}{\eta\psi} < \frac{2a\left(1+\sqrt{1-\eta}\right)}{\eta\psi} := \tilde{r}_2. \tag{28}$$

[22] In environmental economics contexts, e.g., on greenhouse gas emission regulation problems, there is some existing work with dynamic regulation using discrete-time modeling, see for example Buccella et al. (2024).

The following dynamic scenarios occur:

(A) *For* $0 < r < \frac{a}{\psi}$, *the extinction equilibrium* $\bar{X}_1^{CN}$ *is asymptotically stable,* $\bar{X}_2^{CN}$ *is not an equilibrium of the model, equilibrium* $\bar{X}_1^{MSY}$ *is unstable while the equilibrium of partial MSY,* $\bar{X}_2^{MSY}$, *is asymptotically stable with basin of attraction* $\mathcal{B}\left(\bar{X}_2^{MSY}\right) = \left(\bar{X}_1^{MSY}, \frac{K\left(2+r\left(1+\sqrt{1-\eta}\right)\right)}{2r}\right]$, *all the other orbits converge to* $\bar{X}_1^{CN}$;

(B) *At* $r = \frac{a}{\psi}$, *a transcritical bifurcation occurs at which* $\bar{X}_1^{CN}$ *and* $\bar{X}_2^{CN}$ *merge, the global dynamics of the model is as in point (i);*

(C) *For* $\frac{a}{\psi} < r < \tilde{r}_1$, $\bar{X}_1^{CN}$, $\bar{X}_2^{CN}$, $\bar{X}_1^{MSY}$ *and* $\bar{X}_2^{MSY}$ *are equilibria of the model,* $\bar{X}_1^{CN}$ *and* $\bar{X}_1^{MSY}$ *are unstable,* $\bar{X}_2^{MSY}$ *and* $\bar{X}_2^{CN}$ *are asymptotically stable with basins of attraction* $\mathcal{B}\left(\bar{X}_2^{MSY}\right) = \left(\bar{X}_1^{MSY}, \frac{K\left(2+r\left(1+\sqrt{1-\eta}\right)\right)}{2r}\right]$ *and* $\mathcal{B}\left(\bar{X}_2^{CN}\right) = \left(0, \bar{X}_1^{MSY}\right) \cup \left(\frac{K\left(2+r\left(1+\sqrt{1-\eta}\right)\right)}{2r}, \frac{K\left(1+r+\sqrt{1+r(2+r(1-\eta))}\right)}{2r}\right)$, *respectively;*

(D) *At* $r = \tilde{r}_1$, $\bar{X}_2^{CN}$ *and* $\bar{X}_1^{MSY}$ *merge and collide with the kink point* $\tilde{X}$;

(E) *For* $\tilde{r}_1 < r < \tilde{r}_2$, $\bar{X}_1^{CN}$ *and* $\bar{X}_2^{MSY}$ *are the only equilibria,* $\bar{X}_1^{CN}$ *is unstable, while* $\bar{X}_2^{MSY}$ *is asymptotically stable for* $r < \frac{2}{\sqrt{1-\eta}}$, *undergoes a flip bifurcation at* $r = \frac{2}{\sqrt{1-\eta}}$ *and it is unstable for* $r > \frac{2}{\sqrt{1-\eta}}$;

(F) *At* $r = \tilde{r}_2$, *equilibria* $\bar{X}_2^{CN}$ *and* $\bar{X}_2^{MSY}$ *merge and collide with the kink point* $\tilde{X}$, *for* $r > \tilde{r}_2$, $\bar{X}_2^{CN}$ *becomes again feasible (real) while* $\bar{X}_2^{MSY}$ *becomes unfeasible (virtual);*

(G) *For* $\tilde{r}_2 < r < 2 + \frac{a}{\psi}$, *equilibrium* $\bar{X}_2^{CN}$ *is asymptotically stable with basin of attraction*

$$\mathcal{B}\left(\bar{X}_2^{CN}\right) = \left(0, \max\left\{K\frac{1+r-\frac{a}{\psi}}{r}, \frac{K\left(1+r+\sqrt{1+r\left(2+r\left(1-\eta\right)\right)}\right)}{2r}\right\}\right); \quad (29)$$

(H) *At* $r = 2 + \frac{a}{\psi}$, *equilibrium* $\bar{X}_2^{CN}$ *loses stability through a flip (period-doubling) bifurcation;*

(I) *For* $2 + \frac{a}{\psi} < r \leq 3 + \frac{a}{\psi}$, *equilibria* $\bar{X}_{1,2}^{CN}$ *are unstable and orbits in* $\left(0, \max\left\{\frac{K\left(1+r-\frac{a}{\psi}\right)}{r}; \frac{K\left(1+r+\sqrt{1+r(2+r(1-\eta))}\right)}{2r}\right\}\right)$, *except for* $\bar{X}_2^{CN}$, *are attracted to either periodic or chaotic attractors;*

(J) *For* $r > 3 + \frac{a}{\psi}$, *resource extinction occurs in finite time.*

The results in Proposition 7 indicate that fixing a quota at a fraction $\eta \in (0, 1)$ of the MSY leads to the equilibrium of partial MSY being asymptotically stable. On the other hand, the equilibrium of MSY is asymptotically stable only on the right-hand side as shown in Proposition 5. Moreover, the basin of attraction of the equilibrium of partial MSY includes the basin of attraction of the equilibrium of MSY. Therefore, by harvesting only a fraction of the MSY, the risk of extinction associated with maximum sustainable catch is reduced. These advantages are inherited from the fishing quota system. When the quota is fixed as a fraction of the MSY under full compliance, we observe an increase in the basin of attraction of the non-over-exploitation equilibrium. This is observable in the results of Proposition 7 and in the numerical examples of Figs. A.9 and A.10.

The numerical example of Fig. A.9 is obtained using the same parameters and the same map as in Fig. 1(c), but with partial MSY instead of MSY, which means assuming $\eta<1$ instead of $\eta=1$. Comparing Fig. A.9(a) obtained with $\eta=0.9$ with Fig. 1(c), we observe that in Fig. A.9(a) the level of harvesting is reduced and the equilibrium of MSY, $\bar{X}^{MSY}$, is substituted with the equilibrium of partial MSY, $\bar{X}_2^{MSY}$. This equilibrium implies less harvesting and is asymptotically stable, while $\bar{X}^{MSY}$ is only asymptotically stable on the right-hand side. Moreover, the basin of attraction of $\bar{X}^{MSY}$ is a subset of the one of $\bar{X}_2^{MSY}$. Hence, negative shocks at the equilibrium of partial MSY may be absorbed and extinction avoided. Nevertheless, orbits leading to extinction still exist, see the magenta and cyan trajectories in Fig. A.9(a). Reducing η, e.g. $\eta=0.75$, the basin of attraction of $\bar{X}_2^{MSY}$ increases further and the magenta trajectory does not lead to extinction anymore, as shown in Fig. A.9(b). Further reduction of η, e.g. $\eta=0.5$, results in the cyan trajectory not leading to extinction anymore as it is absorbed by the basin of attraction of $\bar{X}_2^{MSY}$ that is further increased, see Fig. A.9(c).

Another issue related to the fishing quota system with quota fixed at the MSY is the coexistence of the equilibrium of MSY with an equilibrium of overexploitation. It is the case of Fig. 2(c), where the fishing quota system only partially reduces the risk of overexploitation. To further attenuate this risk, we can set the quota to a fraction of the MSY as indicated by the results of Proposition 7 and by Fig. A.10. Figure A.10 is obtained by applying the same map and the same parameters as in Fig. 2(c) but fixing the quota at a fraction η, lower than one, of the MSY. Comparing Fig. A.10(a), where $\eta=0.9$, with Fig. 2(c), we observe that the stable equilibrium of partial MSY, $\bar{X}_2^{MSY}$, substitutes the right-hand (only) stable equilibrium of MSY, $\bar{X}^{MSY}$. We also observe that the basin of attraction of $\bar{X}_2^{MSY}$ includes the one of $\bar{X}^{MSY}$. Nevertheless, orbits converging to an equilibrium of overexploitation persist, see, e.g., the magenta and cyan trajectories in Fig. A.10(a). The risk of overexploitation decreases as we reduce η, e.g., in Fig. A.10(b) where $\eta=0.75$, we observe that the basin of attraction of $\bar{X}_2^{MSY}$ is larger and the magenta trajectory does not lead anymore to an equilibrium of overexploitation. Further reducing η, e.g. $\eta=0.5$, eliminates the equilibrium of overexploitation, and the equilibrium of partial MSY, $\bar{X}_2^{MSY}$, remains the only stable equi-

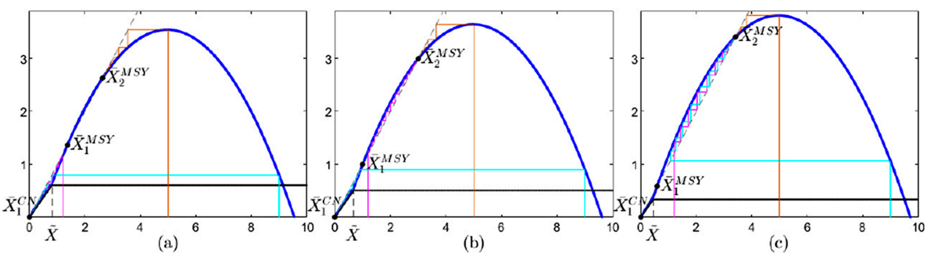

Fig. A.9 Growth under map (8) with harvesting: Panel (**a**) $\eta=0.9$; Panel (**b**) $\eta=0.75$; Panel (**c**) $\eta=0.5$. The dashed line represents biomass X, and the solid black line is harvesting. Staircase diagram of the trajectory starting at $X_0 = 1.2$ is in magenta. Staircase diagram of the trajectory starting at $X_0=5$ is in brown. Staircase diagram of the trajectory starting at $X_0=9$ is in cyan. Equilibria are intersections between the curve of growth and the dashed line and are spotted by a black dot. Remaining parameters as in Fig. 1

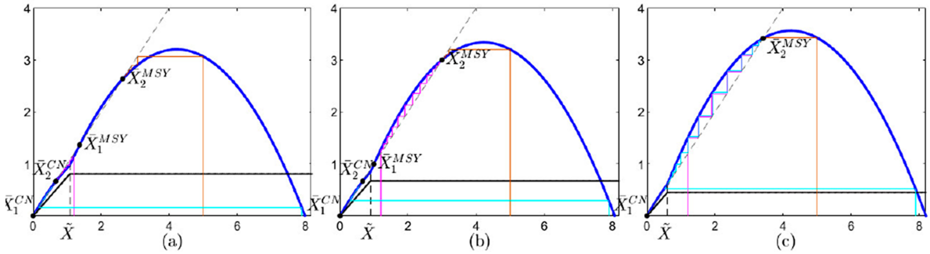

Fig. A.10 Growth under map (8) with harvesting: Panel (**a**) η=0.9; Panel (**b**) η=0.75; Panel (**c**) η=0.5. The dashed line represents biomass X, and the solid black line is harvesting. Staircase diagram of the trajectory starting at $X_0 = 1.2$ is in magenta. Staircase diagram of the trajectory starting at X_0=5 is in brown. Staircase diagram of the trajectory starting at $X_0 = 7.9$ is in cyan. Equilibria are intersections between the curve of growth and the dashed line and are spotted by a black dot. Remaining parameters as in Fig. 2

librium. It is the case of Fig. A.10(c), where even the cyan trajectory converges to $\bar{X}_2^{MSY}$. In this case, the risk of extinction is limited to natural extinction due to overcrowding.

Appendix B. Technical Proofs

Proof of Property 1. By definition $q^{CN}(X) \geq q^{Q}(X)$. Then, properties (A) and (B) follow from Assumption 1(C). From Assumption 1(A) and 1(B) and from the Weierstrass Theorem, we have that a function *f* exists such that $f(q_2; X)$ is the best-reply to q_2. Moreover, by Assumptions 1(A) and by Fermat's Theorem (on stationary points) we have that $f(q_2; X)$ has to be the implicit solution of the equation $\frac{\partial \pi_1}{\partial q_1}(f(q_2; X), q_2; X) = 0$, while by Assumptions 1(A) and 1(B) and by the Implicit Function Theorem, we have that

$$\frac{\partial f}{\partial q_2}(q_2; X) = -\frac{\frac{\partial^2 \pi_1}{\partial q_1 \partial q_2}(f(q_2; X), q_2; X)}{\frac{\partial^2 \pi_1}{\partial q_1^2}(f(q_2; X), q_2; X)}. \tag{30}$$

By Assumption 1(B), the denominator of the right-hand-side of (30) is negative while by Assumption 1(C) the numerator of the right-hand-side of (30) is non-positive. It follows that *f* is decreasing. Then, $f\left(q^{Q}(X); X\right) \geq f\left(q^{CN}(X); X\right)$ since $q^{CN}(X) \geq q^{Q}(X)$. Moreover, $q^{CN}(X) = f\left(q^{CN}(X); X\right)$ from the definition of Cournot-Nash equilibrium. Hence, $f\left(q^{Q}(X); X\right) \geq q^{CN}(X) \geq q^{Q}(X)$. It follows that $\exists \gamma \in (0, 1)$ such that $q^{CN}(X) = \gamma f\left(q^{Q}(X); X\right) + (1-\gamma) q^{Q}(X)$. From Assumption 1(B), we have that:

$$\pi\left(q^{CN}, q^{Q}; X\right) \geq \gamma \pi\left(f\left(q^{Q}(X); X\right), q^{Q}; X\right) + (1-\gamma)\pi\left(q^{Q}, q^{Q}; X\right). \tag{31}$$

By definition of f we have that $\pi\left(f\left(q^{Q}(X); X\right), q^{Q}; X\right) \geq \pi\left(q, q^{Q}; X\right)$ for all q. Then, property (C) follows from inequality (32). Property (D) follows by noting that $\pi\left(q^{CN}(X), q^{CN}(X); X\right) \geq \pi\left(q^{Q}(X), q^{CN}(X); X\right)$ by definition of Cournot-Nash equilibrium and $\pi\left(q^{CN}(X), q^{CN}(X); X\right) > \pi\left(q^{Q}(X), q^{CN}(X); X\right)$ when

$q^{CN}(X) \neq q^{Q}(X)$ by Assumption 1(B) of strictly concavity. Finally, Property (E) follows from Properties (A) and (C).

Proof of Proposition 1. Setting $\mu = 0$, the game becomes

	Column Player		
	Strategies	q^Q	q^{CN}
Row Player	q^Q	$\left(\pi\left(q^Q, q^Q\right), \pi\left(q^Q, q^Q\right)\right)$	$\left(\pi\left(q^Q, q^{CN}\right), \pi\left(q^{CN}, q^Q\right)\right)$
	q^{CN}	$\left(\pi\left(q^{CN}, q^Q\right), \pi\left(q^Q, q^{CN}\right)\right)$	$\left(\pi\left(q^{CN}, q^{CN}\right), \pi\left(q^{CN}, q^{CN}\right)\right)$

where we have dropped the dependence on X of q^Q, q^{CN} and π for the sake of notation simplicity and without loss of generality. Being q^{CN} the harvesting at the Cournot-Nash equilibrium, we have that $\left(q^{CN}, q^{CN}\right)$ is always a Nash equilibrium of the game. By definition of Nash equilibrium, $\left(q^Q, q^Q\right)$ is a Nash equilibrium if and only if $\pi\left(q^Q, q^Q\right) = \pi\left(q^{CN}, q^Q\right)$. By Property 1-(C), this implies $q^{CN} = q^Q$. The remaining part of the proposition follows from the definition of the prisoner's delight (dilemma) game.

Proof of Proposition 2. By assumption, $1 \geq \lambda_2 > \lambda_1 \geq 0$, $\mu \geq 0$ and the game is symmetric. Therefore, the game becomes

	Column Player		
	Strategies	q^Q	q^{CN}
Row Player	q^Q	$\left(\pi\left(q^Q, q^Q\right), \pi\left(q^Q, q^Q\right)\right)$	$\left(\pi\left(q^Q, q^{CN}\right), \pi\left(q^{CN}, q^Q\right) - \lambda_1\mu\right)$
	q^{CN}	$\left(\pi\left(q^{CN}, q^Q\right) - \lambda_1\mu, \pi\left(q^Q, q^{CN}\right)\right)$	$\left(\pi\left(q^{CN}, q^{CN}\right) - \lambda_2\mu, \pi\left(q^{CN}, q^{CN}\right) - \lambda_2\mu\right)$

where we have dropped the dependence on X of q^Q, q^{CN} and π for the sake of notation simplicity and without loss of generality. By definition of Nash equilibrium, $\left(q^Q, q^Q\right)$ is a Nash equilibrium if and only if

$$\lambda_1\mu \geq \pi\left(q^{CN}, q^Q\right) - \pi\left(q^Q, q^Q\right) = \mu_1; \tag{32}$$

$\left(q^{CN}, q^{CN}\right)$ is a Nash equilibrium if and only if

$$\lambda_2\mu \leq \pi\left(q^{CN}, q^{CN}\right) - \pi\left(q^Q, q^{CN}\right) = \mu_2; \tag{33}$$

$\left(q^{CN}, q^Q\right)$ is a Nash equilibrium if and only if

$$\mu_2 = \pi\left(q^{CN}, q^{CN}\right) - \pi\left(q^Q, q^{CN}\right) \leq \lambda_2\mu \text{ and } \lambda_1\mu \leq \pi\left(q^{CN}, q^Q\right) - \pi\left(q^Q, q^Q\right) = \mu_1. \tag{34}$$

Since the game is symmetric, $\left(q^Q, q^{CN}\right)$ is a Nash equilibrium if and only if $\left(q^{CN}, q^Q\right)$ is a Nash equilibrium. Moreover,

$$\mu \gtrless \mu_3 \Leftrightarrow \pi\left(q^Q, q^Q\right) \gtrless \pi\left(q^{CN}, q^{CN}\right) - \lambda_2\mu. \tag{35}$$

It follows that $\left(q^{CN}, q^{CN}\right)$ is more (less) profitable than $\left(q^{Q}, q^{Q}\right)$ when $\lambda_2\mu < \mu_3$ $(\lambda_2\mu > \mu_3)$. By Property 1, we have that

$$\pi\left(q^{CN}, q^{Q}\right) > \pi\left(q^{Q}, q^{Q}\right) \quad \text{and} \quad \pi\left(q^{CN}, q^{CN}\right) > \pi\left(q^{Q}, q^{CN}\right). \tag{36}$$

Therefore, $\mu_1, \mu_2 > 0$. This completes the proof.

Proof of Proposition 3. By assumption, $\lambda = 1$, $\mu \geq 0$ and the game is symmetric. Therefore, the game becomes

		Column Player	
	Strategies	q^{Q}	q^{CN}
Row Player	q^{Q}	$\left(\pi\left(q^{Q}, q^{Q}\right), \pi\left(q^{Q}, q^{Q}\right)\right)$	$\left(\pi\left(q^{Q}, q^{CN}\right), \pi\left(q^{CN}, q^{Q}\right) - \mu\right)$
	q^{CN}	$\left(\pi\left(q^{CN}, q^{Q}\right) - \mu, \pi\left(q^{Q}, q^{CN}\right)\right)$	$\left(\pi\left(q^{CN}, q^{CN}\right) - \mu, \pi\left(q^{CN}, q^{CN}\right) - \mu\right)$

where we have dropped the dependence on X of q^{Q}, q^{CN} and π for the sake of notation simplicity and without loss of generality. By definition of Nash equilibrium, $\left(q^{Q}, q^{Q}\right)$ is a Nash equilibrium if and only if

$$\mu \geq \pi\left(q^{CN}, q^{Q}\right) - \pi\left(q^{Q}, q^{Q}\right) = \mu_1; \tag{37}$$

$\left(q^{CN}, q^{CN}\right)$ is a Nash equilibrium if and only if

$$\mu \leq \pi\left(q^{CN}, q^{CN}\right) - \pi\left(q^{Q}, q^{CN}\right) = \mu_2; \tag{38}$$

$\left(q^{CN}, q^{Q}\right)$ is a Nash equilibrium if and only if

$$\mu_2 = \pi\left(q^{CN}, q^{CN}\right) - \pi\left(q^{Q}, q^{CN}\right) \leq \mu \leq \pi\left(q^{CN}, q^{Q}\right) - \pi\left(q^{Q}, q^{Q}\right) = \mu_1. \tag{39}$$

Since the game is symmetric, $\left(q^{Q}, q^{CN}\right)$ is a Nash equilibrium if and only if $\left(q^{CN}, q^{Q}\right)$ is a Nash equilibrium. Moreover,

$$\mu \gtrless \mu_3 \Leftrightarrow \pi\left(q^{Q}, q^{Q}\right) \gtrless \pi\left(q^{CN}, q^{CN}\right) - \mu. \tag{40}$$

It follows that $\left(q^{CN}, q^{CN}\right)$ is more (less) profitable than $\left(q^{Q}, q^{Q}\right)$ when $\mu < \mu_3$ $(\mu > \mu_3)$. By property 1, we have that

$$\pi\left(q^{CN}, q^{Q}\right) > \pi\left(q^{Q}, q^{Q}\right) \quad \text{and} \quad \pi\left(q^{CN}, q^{CN}\right) > \pi\left(q^{Q}, q^{CN}\right). \tag{41}$$

Therefore, $\mu_1, \mu_2 > 0$. This completes the proof.

Proof of Proposition 4. Let us recall the global dynamics of the standard logistic map $x_{t+1} = \hat{r}x_t\left(1 - x_t\right)$, see, e.g., Feigenbaum (1978), Devaney (1989), Phatak and Suresh Rao (1995) and Murray (2003). It is well-known that this map has two equilibria for $\hat{r} > 0$, that is, $x_1 = 0$ and $x_2 = 1 - 1/\hat{r}$. For $\hat{r} \in (0, 1)$, x_2 is unstable, $x_1 = 0$ is asymptotically stable, with $\mathcal{B}\left(x_1\right) = \left(x_2, 1/\hat{r}\right)$ and outside $\left[x_2, 1/\hat{r}\right]$ orbits are attracted to $-\infty$. For $\hat{r} = 1$, we have a transcritical bifurcation, with the two equilibria that merge in zero, that is $x_1 = x_2 = 0$. Moreover, $\mathcal{B}(0) = [0, 1]$, outside $\mathcal{B}(0)$ orbits are attracted to $-\infty$. For $\hat{r} \in (1, 3)$, x_1 is unstable, x_2 is asymptotically stable with $\mathcal{B}(x_2) = (0, 1)$. Orbits outside $[0, 1]$ are attracted to $-\infty$. At $\hat{r} = 3$, the equilibrium x_2 undergoes a flip (or period-doubling) bifurcation and a stable 2-cycle appears and persists stable for $\hat{r} \in \left(3, 1 + \sqrt{6}\right)$. At $\hat{r} = 1 + \sqrt{6}$, the 2-cycle undergoes a period-doubling bifurcation itself becoming unstable and a stable 4-cycle appears. By further increasing $\hat{r}$, cycles of period $2, 4, 8, 16, \ldots, 2^k, 2^{k+1}, \ldots$ appear through a so-called cascade of period-doubling bifurcations. These cycles are stable when appear through a period-doubling bifurcation and become unstable (but persist) at the next period-doubling bifurcation. For $\hat{r} = \hat{r}_\infty \approx 3.56994$, so-called *Feigenbaum point*, all the infinite 2^k-cycles are originated and are now unstable. For $\hat{r} \in (\hat{r}_\infty, 4)$, all trajectories are chaotic. At $\hat{r} = 4$, a final bifurcation occurs with *aperiodic* (chaotic) trajectories. For $\hat{r} > 4$, almost all trajectories diverge to $-\infty$. Defining $x = X\frac{r}{K(1+r)}$, the map in (8) with $H(X) = 0$ becomes the standard logistic map $x_{t+1} = x_t \hat{r}(1 - x_t)$, where $\hat{r} = 1 + r$. Hence, point (i) follows. Regarding the map with MSY harvesting, let us operate the change of variable $Y = X - b$, where $b = K\left(1 - \sqrt{1-\eta}\right)/2$. Then, we obtain a map that is equivalent to map (8) where $H(X) = 0$, parameter r is substituted with parameter $\bar{r} = r\sqrt{1-\eta}$ and parameter K is replaced with parameter $\bar{K} = K\sqrt{1-\eta}$. Hence, Point (ii) follows. Define $\tilde{r} = r - \frac{a}{\psi}$ and $\tilde{K} = \frac{K\tilde{r}}{r}$. Map (8) with Cournot-Nash harvesting can be rewritten as a map without harvesting, with rescaled growth rate $\tilde{r}$ and carrying capacity $\tilde{K}$. Hence, Point (iii) follows.

Proof of Corollary 1. For $a > r\psi$, note that:

$$\bar{X}_2^{MSY} = \frac{K}{2}\left(1 + \sqrt{1-\eta}\right) < \bar{X}_2^{CN} = K\left(1 - \frac{a}{r\psi}\right) \Leftrightarrow r\psi\frac{(1 + \sqrt{1-\eta})}{2} > a. \quad (42)$$

Moreover, by Proposition 4 we have that

$$\mathcal{B}\left(\bar{X}_2^{CN}\right) = \left(0, K\frac{1 + r - \frac{a}{\psi}}{r}\right) \quad \text{and} \quad \mathcal{B}\left(\bar{X}_2^{MSY}\right) = \left(\bar{X}_1^{MSY}, K\frac{2 + r\,(1+\sqrt{1-\eta})}{2r}\right). \quad (43)$$

Since $\bar{X}_1^{MSY} > 0$ for all $r>0$, $\mathcal{B}\left(\bar{X}_2^{CN}\right) \subset \mathcal{B}\left(\bar{X}_2^{MSY}\right)$ is not possible while we have that $\mathcal{B}\left(\bar{X}_2^{MSY}\right) \subset \mathcal{B}\left(\bar{X}_2^{CN}\right)$ when

$$K\frac{1 + r - \frac{a}{\psi}}{r} > K\frac{2 + r\,(1+\sqrt{1-\eta})}{2r} \quad (44)$$

that is, if and only if $a < \tilde{a}$.

Proof of Proposition 5. Let us start by noting that the piecewise-smooth map (21) is the logistic map (8) with Cournot-Nash harvesting for $X < \tilde{X}$ and the logistic map (8) with MSY harvesting (η=1) for $X < \tilde{X}$. Therefore, by Proposition 4, the possible equilibria are $\bar{X}_1^{CN} = 0$ and $\bar{X}_2^{CN} = K\left(1 - \frac{a}{r\psi}\right)$ as long as they are in the region $X < \tilde{X}$ and $\bar{X}^{MSY} = \frac{K}{2}$ as long as it is in the region $X > \tilde{X}$. If $\bar{X}_1^{CN} < \tilde{X}$, we say that it is feasible (otherwise virtual). The same for $\bar{X}_2^{CN}$. If $\bar{X}^{MSY} > \tilde{X}$, we say that this equilibrium is feasible (otherwise virtual). As long as an equilibrium is feasible, and it is not a kink point, its local stability properties and bifurcations are the same as those described in Proposition 4. Regarding its basins of attraction, as long as it is contained in its own region of feasibility it is the same as in Proposition 4 with the addition of all its eventual preimages, otherwise it will be determined by analytical considerations in the following of this proof. If an equilibrium is virtual, then it does not exist and orbits cannot end up in it. For these standard properties of piecewise-smooth one-dimensional map we refer to Avrutin et al. (2019). Consider the case $0 < r < \frac{a}{\psi}$. Then, $\bar{X}^{MSY}$ is feasible and $\mathcal{B}\left(\bar{X}^{MSY}\right)$, determined using map (8) with MSY harvesting (η=1), is inside the region where this equilibrium is feasible ($X > \tilde{X}$). All the other orbits in $X > \tilde{X}$ converge in a finite number of iterations in the region $X < \bar{X}^{MSY}$, where there is monotonic convergence to $-\infty$ if the map (8) with MSY harvesting (η=1) applies. Therefore, all these orbits enter in a finite number of iterations the region $X < \tilde{X}$, where map (8) with Cournot-Nash harvesting applies and where there is monotonic convergence to $\bar{X}_1^{CN}$. It follows that all trajectories outside $\mathcal{B}\left(\bar{X}^{MSY}\right)$ are attracted to $\bar{X}_1^{CN}$. This completes the proof of Proposition 5-(A). At $r = \frac{a}{\psi}$ the same conditions that characterize Proposition 5-(A) are satisfied with the only novelty that $\bar{X}_1^{CN} = \bar{X}_2^{CN}$. The merging of the two equilibria occurs inside the region where the logistic map (8) with Cournot-Nash harvesting applies. Hence, as stated in Proposition 4-(C) the merging occurs because of a transcritical bifurcation, which proves Proposition 5-(B). For $\frac{a}{\psi} < r < \frac{2a}{\psi}$, we have that equilibria $\bar{X}_{1,2}^{CN}$ and $\bar{X}^{MSY}$ are all feasible and are not a kink point, therefore their local stability properties in Proposition 4 applies. Moreover, according to Proposition 4, we have that $\mathcal{B}\left(\bar{X}^{MSY}\right) = \left[\bar{X}^{MSY}, \frac{K(2+r)}{2r}\right]$ which is contained in the region where the map (8) with MSY harvesting (η=1) apply. Moreover, in $\left(\tilde{X}, \bar{X}^{MSY}\right) \cup \left(\frac{K(2+r)}{2r}, \frac{K\left(1+r+\sqrt{1+2r}\right)}{2r}\right)$ we have that map (8) with MSY harvesting (η=1) applies, is always positive and, by Proposition 4, all orbits are attracted to $-\infty$ if map (8) with MSY harvesting (η=1) applied everywhere. Therefore, any of these orbits enter $\left(0, \tilde{X}\right)$ in a finite number of iterations. There, map (8) with Cournot-Nash harvesting applies and is always increasing for $\frac{a}{\psi} < r < \frac{2a}{\psi}$. Indeed, it is a concave parabola with positive derivative in $\tilde{X}$, which is given by $1 + r - \frac{r^2\psi}{2a} - \frac{a}{\psi}$. Set $z = a/\psi$, the derivative can be rewritten as $2z + 2zr - r^2 - 2z^2$. Hence it is a parabola in r. Moreover, in r=z it values $z\left(2 - z\right)$. Therefore it is positive for z<2, and in $r = 2z$ it values $2z\left(1 - z\right)$, hence positive for z<1. This implies that it is positive for all $r \in (z, 2z)$ as long as z<1. Since z<1 by assumption, for $r \in (z, 2z)$ we have that the derivative of the map (8) with Cournot-Nash harvesting is positive in $\tilde{X}$. Then, orbits are monotonic in $\left(0, \tilde{X}\right)$, and by Proposition 4 these orbits are attracted to $\bar{X}_2^{CN}$. This proves Proposition 5-(C). At $r = 2a/\psi$, it is $\bar{X}^{MSY} = \bar{X}_2^{CN} = \tilde{X}$, whence the kink point $\tilde{X}$ is also an equilibrium and attracts all orbits that $\bar{X}^{MSY}$ and $\bar{X}_2^{CN}$ attract. This proves Proposition 5-(D). For $r > 2a/\psi$, we have always $\bar{X}_{1,2}^{CN} < \tilde{X}$ and by Proposition 4 $\bar{X}_1^{CN}$ is unstable while $\bar{X}_2^{CN}$ is asymptotically stable. Moreover, for $\frac{2a}{\psi} < r < 2 + \frac{a}{\psi}$, we have that $\bar{X}^{MSY} < \tilde{X}$. Therefore, $\bar{X}^{MSY}$ is not a feasible equilibrium of the model. This proves Proposition 5-(E) except for the basin of attraction of $\bar{X}_2^{CN}$ when $r \geq \frac{2a}{\psi} + 2\sqrt{\frac{a}{\psi}}$. At $r = \frac{2a}{\psi} + 2\sqrt{\frac{a}{\psi}}$, we have that the kink point $\tilde{X}$ is also the point $K\frac{1+r-\frac{a}{\psi}}{r}$ at which the concave parabola representing the map (8) with

Cournot-Nash harvesting values zero. Therefore, $\left(0, K\frac{1+r-\frac{a}{\psi}}{r}\right)$ is an invariant set where the dynamics is given by the smooth map (8) with Cournot-Nash harvesting and any point inside this region does not have any preimages outside, where the map takes either zero or negative values. The same applies for $r > \frac{2a}{\psi} + 2\sqrt{\frac{a}{\psi}}$. The results of Proposition 4-(C) in this invariant set are then valid. Indeed, two maps that are equal in a subset (interval) $\mathcal{S}$ of their state space, which is also a (forward) invariant set ($\mathcal{S}$ is a (forward) invariant set for a map when $x_t \in \mathcal{S}$ implies $x_{t+1} \in \mathcal{S}$ for all $t \in \mathbb{N}$), have in $\mathcal{S}$ the same dynamics. This proves the remaining part of Proposition 5-(E) and proves Proposition 5-(F), (G), (H).

Proof of Proposition 6. Note that for $X \leq X^+$, map (25) coincides with map (21), therefore equilibria and their stability are as in Proposition 5. Moreover, for $X > X^+$ map (25) is the same as map (8) with Cournot-Nash harvesting, then equilibria and their stability are as in Proposition 4-(iii). It follows that the only element to verify to complete the investigation of the equilibria is their feasibility. Differently from Proposition 5, we have that $\bar{X}_2^{CN}$ is feasible even when $\bar{X}_2^{CN} > X^+$ while $\bar{X}^{MSY}$ is not feasible when $\bar{X}^{MSY} > X^+$. Note that, $\bar{X}^{MSY} > X^+$ if and only if

$$\left(\psi a K r + 8\psi\mu - 2a^2 K\right)^2 > 16aKr\mu\psi^2 + 64\mu^2\psi^2 \text{ and } \psi aKr + 8\psi\mu - 2a^2K < 0, \quad (45)$$

which is equivalent to condition

$$\mu < \min\left\{\frac{K(2a - r\psi)^2}{32\psi}; aK\frac{2a - \psi r}{8\psi}\right\}. \quad (46)$$

Note that $\frac{K(2a-\psi r)^2}{32\psi} < aK\frac{2a-\psi r}{8\psi}$ for $r < 2a/\psi$ and $\frac{K(2a-\psi r)^2}{32\psi} = aK\frac{2a-\psi r}{8\psi} = 0$ for $r = 2a/\psi$ and $aK\frac{2a-\psi r}{8\psi} < 0$ for $r > 2a/\psi$. Hence, for $r > 2a/\psi$, it always holds that $\bar{X}^{MSY} < X^+$, while for $r < 2a/\psi$ we have $\bar{X}^{MSY} < X^+$ for $\mu < \tilde{\mu}$. Note that $\bar{X}_2^{CN} > X^+$ if and only if

$$8aKr\psi^2\left(8a^2 - 8ar\psi - \psi^2r^2\right)\mu > K^2a^2\left(-16a^2r^2\psi^2 - 16a^4 - \psi^4r^4 + 32a^3r\psi + 8ar^2\psi^2\left(r\psi - 1\right)\right) \quad (47)$$

and $\frac{4a^2K(r\psi-a)-aKr^2\psi^2}{8r\psi^2} > \mu$. The term $\frac{4a^2K(r\psi-a)-aKr^2\psi^2}{8r\psi^2}$ is never positive and values 0 only for $r = 2a/\psi$. Hence, $\bar{X}_2^{CN} > X^+$ is never possible. Regarding the basin of attraction, as we know from Proposition 4-(ii), harvesting always the quota (dynamical system (8) with MSY) we have $\mathcal{B}\left(\bar{X}^{MSY}\right) = \left[\bar{X}^{MSY}, \frac{K(2+r)}{2r}\right]$. Moreover, for $X < X^+$ map (25) is the same as map (21). Then, from Proposition 5, we know that there are not other points in the region $X < X^+$ that belongs to the basin of attraction of $\bar{X}^{MSY}$ except for those in $\left[\bar{X}^{MSY}, \frac{K(2+r)}{2r}\right]$. By definition of map (25), we also have that for $X > X^+$ fishers do not comply with the regulation, therefore they harvest an amount larger than the quota. Then, each X that is greater than X^+ is mapped into a point that is lower than the point at which the same X is mapped by dynamical system (8) with MSY. It follows that, either $X^+ > \frac{K(2+r)}{2r}$ and $\mathcal{B}\left(\bar{X}^{MSY}\right) = \left[\bar{X}^{MSY}, \frac{K(2+r)}{2r}\right]$ or the basin of attraction of $\bar{X}^{MSY}$ is a subset of $\left[\bar{X}^{MSY}, \frac{K(2+r)}{2r}\right]$. Note that $X^+ < \frac{K(2+r)}{2r}$ if and only if $\mu < \hat{\mu}$. To complete the proof note that for $r \geq \frac{2a}{\psi} + 2\sqrt{\frac{a}{\psi}}$, in any point in which the map (25) is positive, we have that it is equivalent to map (21). Therefore, the same results as in Proposition 5 apply.

Proof of Proposition 7. Note that $\bar{X}_1^{MSY} < \tilde{X}$ if and only if $\frac{a}{\psi}\frac{1-\sqrt{1-\eta}}{\eta} < r$, $\bar{X}_2^{MSY} > \bar{X}_1^{MSY}$ and $\frac{1-\sqrt{1-\eta}}{\eta} > 1$ since $\eta \in (0,1)$ by assumption. Therefore, $\bar{X}_{1,2}^{MSY}$ are feasible $\left(> \tilde{X}\right)$ for $r < \frac{a}{\psi}$. The remaining parts of Proposition 7-(A) follow from Proposition 5-(A) and Proposition 4. Note that η impacts only on map (8) with MSY harvesting. Therefore, Proposition 7-(B) follows from Proposition 5-(B). For $\frac{a}{\psi} < r < \tilde{r}_1$, we have $\bar{X}_1^{CN} < \bar{X}_2^{CN} < \tilde{X} < \bar{X}_1^{MSY} < \bar{X}_2^{MSY}$. Then, all four equilibria are feasible and their local stability follows from Proposition 4. Moreover, note that all points in the interval $\mathcal{B}\left(\bar{X}_2^{MSY}\right)$ are greater than $\tilde{X}$, therefore $\mathcal{B}\left(\bar{X}_2^{MSY}\right)$ is basin of attraction of $\bar{X}_2^{MSY}$, as it follows from Proposition 4 and by noting that $\mathcal{B}\left(\bar{X}_2^{MSY}\right)$ does not have preimages. The basin of attraction of $\bar{X}_2^{CN}$ follows by noting that (for $a/\psi < r < \tilde{r}_1$) all points in the region where the map is positive and outside $\mathcal{B}\left(\bar{X}_2^{MSY}\right)$ remain positive forever when iterated. Then, they must converge to an attractor. Then, they either converge to the stable equilibrium $\bar{X}_2^{CN}$ or to a periodic or chaotic attractor. However, the presence of periodic or chaotic attractors implies that a point greater than $\tilde{X}$ has preimages in the region $\left(0, \tilde{X}\right)$. This is not possible as the map in $\left(0, \tilde{X}\right)$ is increasing (as already shown in the proof of Proposition 5) with trajectories that monotonically converge to $\bar{X}_2^{CN}$. Then, they converge to $\bar{X}_2^{CN}$. This completes the proof of Proposition 7-(C). For $r = \tilde{r}_1$, we have $\bar{X}_2^{CN} = \tilde{X} = \bar{X}_1^{MSY}$, which proves Proposition 7-(D). For $\tilde{r}_1 < r < \tilde{r}_2$ note that $\bar{X}_2^{CN} > \tilde{X} > \bar{X}_1^{MSY}$, while $\bar{X}_2^{MSY} > \tilde{X}$. Therefore, equilibria are $\bar{X}_2^{MSY}$ and $\bar{X}_1^{CN}$ and Proposition 7-(E) follows from Proposition 4. For $r = \tilde{r}_2$, we have $\bar{X}_2^{MSY} = \tilde{X} = \bar{X}_2^{CN}$, while for $r > \tilde{r}_2$, we have $\bar{X}_2^{MSY}, \bar{X}_2^{CN} < \tilde{X}$. Therefore, $\bar{X}_1^{MSY} < \tilde{X}$ and $\bar{X}_{1,2}^{MSY}$ are not feasible while $\bar{X}_2^{CN}$ is feasible. This proves Proposition 7-(F), while Proposition 7-(G), (H), (I) and (L) follows from Proposition 4 except for the basins of attraction. Regarding the basin of attraction of $\bar{X}_2^{CN}$ in Proposition 7-(G), it follows by noting that a point where the map is positive is still mapped to a point where the map is positive and by noting that in the region where the map is positive, there is only one invariant set, which is the stable equilibrium $\bar{X}_2^{CN}$. Regarding the basin of attraction of periodic or chaotic attractors in Proposition 7-(I), the claim follows by noting that a point where the map is positive is still mapped to a point where the map is positive and by noting that in the region where the map is positive there is only an unstable equilibrium $\bar{X}_2^{CN}$, therefore all the other points must converge to some periodic or chaotic attractor.

Appendix C. Derivation of the Harvesting Function in Case of Partial Compliant Strategy

Consider the harvesting function (24). According to the fishery setup of Sect. 4, we have that:

$$\pi\left(q^{CN}(X), q^{CN}(X); X\right) = \pi\left(q^{CN}(X), q^{Q}(X); X\right) = \frac{a^2 X}{4\psi} \tag{48}$$

and

$$\pi\left(q^{Q}\left(X\right),q^{Q}\left(X\right);X\right)=\pi\left(q^{Q}\left(X\right),q^{CN}\left(X\right);X\right)=\begin{cases}\frac{a^{2}X}{4\psi} & \text{if } \frac{aX}{2\psi}<\frac{\eta Kr}{8}\\ \left(a-\frac{\eta Kr\psi}{8X}\right)\frac{\eta Kr}{8} & \text{if } \frac{aX}{2\psi}>\frac{\eta Kr}{8}\end{cases}. \tag{49}$$

Hence,

$$\mu_{1}=\mu_{2}=\begin{cases}0 & \text{if } \frac{aX}{2\psi}<\frac{\eta Kr}{8}\\ \frac{a^{2}X}{4\psi}-\left(a-\frac{\eta Kr\psi}{8X}\right)\frac{\eta Kr}{8} & \text{if } \frac{aX}{2\psi}>\frac{\eta Kr}{8}\end{cases} \tag{50}$$

and the harvesting function (24) can be rewritten as

$$H\left(X\right)=\begin{cases}2q^{Q}\left(X\right) & \text{if } \mu>\frac{a^{2}X}{4\psi}-\left(a-\frac{\eta Kr\psi}{8X}\right)\frac{\eta Kr}{8} \text{ and } \frac{aX}{2\psi}>\frac{\eta Kr}{8}\\ 2q^{CN}\left(X\right) & \text{otherwise}\end{cases} \tag{51}$$

where $2q^{Q}\left(X\right)=\frac{\eta Kr}{4}$ and $2q^{CN}\left(X\right)=\frac{aX}{\psi}$. Moreover, note that $\mu>\frac{a^{2}X}{4\psi}-\left(a-\frac{\eta Kr\psi}{8X}\right)\frac{\eta Kr}{8}$ if and only if $X^{-}<X<X^{+}$, where

$$X^{\pm}=\frac{\psi\left(aKr\eta+8\mu\right)\pm\psi\sqrt{\left(aKr\eta+8\mu\right)^{2}-a^{2}K^{2}r^{2}\eta^{2}}}{4a^{2}}. \tag{52}$$

Note that $X^{-}<\frac{\eta Kr\psi}{4a}:=\tilde{X}<X^{+}$. Hence, the harvesting function (24) can be rewritten as

$$H\left(X\right)=\begin{cases}\frac{\eta Kr}{4} & \text{if } \tilde{X}<X<X^{+}\\ \frac{aX}{\psi} & \text{otherwise}\end{cases}. \tag{53}$$

Plugging the harvesting function (53) in the dynamics framework for the evolution of the biomass given by (8), we obtain map (25).

As a remark, note that $X^{-}=\frac{\eta Kr\psi}{4a}:=\tilde{X}=X^{+}$ and $2q^{Q}\left(X^{+}\right)=2q^{CN}\left(X^{+}\right)=\frac{\eta Kr}{4}$ when $\mu=0$. Hence, for $\mu=0$ the harvesting function (53) reduces to $2q^{CN}\left(X\right)$ and map (25) becomes the map (8) with Cournot-Nash harvesting.

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Declarations

Conflict of interest The Authors declare no conflict of interest.

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